Edexcel A level Maths Trigonometric identities

Topic assessment

1. Solve the equation $\sin(x+30°) = 2\cos(x+45)°$ for $0° \le x \le 360°$. [5]

2. If $4\cos 2x - 2\cos x + 1 = 0$, find values for x in the range $0° \le x \le 360°$. [6]

3. Write $3\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, and hence solve the equation $3\cos\theta + 5\sin\theta = 2$ in the range $0° \le \theta \le 360°$. [8]

4. Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} \equiv \tan x$. [5]

5. (i) Write sin 3x in terms of sin x. [4]
   (ii) Solve $\sin 3x = \sin^2 x$ for $0° \le x \le 360°$. [5]

6. (i) Prove that $4\left(\sin^4 x + \cos^4 x\right) \equiv \cos 4x + 3$ [6]
   (ii) Solve the equation $\sin^4 x + \cos^4 x = 0.5$ for $0 \le x \le 2\pi$. [4]

7. Find the minimum value of the expression
   $4\cos x - 3\sin x - 4$.
   Give the smallest possible positive value of x for which this minimum value occurs. [7]

Total 50 marks

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Solutions to Topic Assessment

1. $\sin(x+30°)=2\cos(x+45)°$

$\sin x\cos 30°+\cos x\sin 30°=2\cos x\cos 45°-2\sin x\sin 45°$

$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\frac{2\sqrt{2}}{2}\cos x-\frac{2\sqrt{2}}{2}\sin x$

$\sqrt{3}\sin x+\cos x=2\sqrt{2}\cos x-2\sqrt{2}\sin x$

$\left(\sqrt{3}+2\sqrt{2}\right)\sin x=\left(2\sqrt{2}-1\right)\cos x$

$\tan x=\frac{2\sqrt{2}-1}{\sqrt{3}+2\sqrt{2}}$

$x=21.8°$ or $201.8°$

[5]

2. $4\cos 2x-2\cos x+1=0$

$4(2\cos^2 x-1)-2\cos x+1=0$

$8\cos^2 x-4-2\cos x+1=0$

$8\cos^2 x-2\cos x-3=0$

$(4\cos x-3)(2\cos x+1)=0$

$\cos x=\frac{3}{4}$ or $-\frac{1}{2}$

$\cos x=\frac{3}{4}\quad\Rightarrow x=41.4°$ or $318.6°$

$\cos x=-\frac{1}{2}\quad\Rightarrow x=120°$ or $240°$

$x=41.4°$ or $120°$ or $240°$ or $318.6°$.

[6]

3. $3\cos\theta+5\sin\theta=R\cos(\theta-\alpha)=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$

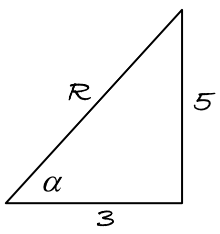
$R\cos\alpha=3$

$R\sin\alpha=5$

$R^2=3^2+5^2=34\quad\Rightarrow R=\sqrt{34}$

$\tan\alpha=\frac{5}{3}\quad\Rightarrow 59.04°$

$3\cos\theta+5\sin\theta=\sqrt{34}\cos(\theta-59.04°)$



$3\cos\theta+5\sin\theta=2$

$\sqrt{34}\cos(\theta-59.04°)=2$

$\cos(\theta-59.04°)=\frac{2}{\sqrt{34}}$

$\theta-59.04°=69.94°$ or $290.06°$

$\theta=129.0°$ or $349.1°$

[8]

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4. $\text{L.H.S.} \equiv \dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1}$

$\equiv \dfrac{\cancel{2}\sin x\cos x + \cancel{2}\sin^2 x}{\cancel{2}\sin x\cos x + \cancel{2}\cos^2 x}$

Using the double angle formula, $1+\cos 2x = 2\cos^2 x$ and $1-\cos 2x = 2\sin^2 x$

$\equiv \dfrac{\sin x(\cancel{\cos x + \sin x})}{\cos x(\cancel{\sin x + \cos x})}$

$\equiv \dfrac{\sin x}{\cos x}$

$\equiv \tan x$

[5]

5. (i) $\sin 3x = \sin(2x + x)$

$= \sin 2x\cos x + \cos 2x\sin x$

$= 2\sin x\cos x\cos x + (1 - 2\sin^2 x)\sin x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

[4]

(ii) $\sin 3x = \sin^2 x$

$3\sin x - 4\sin^3 x = \sin^2 x$

$4\sin^3 x + \sin^2 x - 3\sin x = 0$

$\sin x(4\sin^2 x + \sin x - 3) = 0$

$\sin x(4\sin x - 3)(\sin x + 1) = 0$

$\sin x = 0$ or $\frac{3}{4}$ or -1

$\sin x = 0 \Rightarrow x = 0°$ or $180°$ or $360°$

$\sin x = \frac{3}{4} \Rightarrow x = 48.6°$ or $131.4°$

$\sin x = -1 \Rightarrow x = 270°$

$x = 0°, 48.6°, 131.4°, 180°, 270°, 360°$

[5]

6. (i) $\text{L.H.S.} \equiv 4\left(\sin^4 x + \cos^4 x\right)$

$\equiv \left(2\sin^2 x\right)^2 + \left(2\cos^2 x\right)^2$

$\equiv (1 - \cos 2x)^2 + (1 + \cos 2x)^2$

$\equiv 1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x$

$\equiv 2 + 2\cos^2 2x$

$\equiv 2 + \cos 4x + 1$

$\equiv \cos 4x + 3$

$= \text{R.H.S.}$

[6]

(ii) $\sin^4 x + \cos^4 x = 0.5$

$4(\sin^4 x + \cos^4 x) = 2$

$\cos 4x + 3 = 2$

$\cos 4x = -1$

$4x = \pi, 3\pi, 5\pi, 7\pi$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

[4]

7. $4\cos x - 3\sin x = R\cos(x + \alpha)$

$= R\cos x \cos\alpha - R\sin x \sin\alpha$

$4 = R\cos\alpha$

$3 = R\sin\alpha$

$R^2 = 4^2 + 3^2 = 25 \quad \Rightarrow R = 5$

$\tan\alpha = \frac{3}{4} \quad \Rightarrow 36.9°$

$4\cos x - 3\sin x - 4 = 5\cos(x + 36.9°) - 4$

The minimum value of the expression is $-5 - 4 = -9$.

This occurs when $x + 36.9° = 180° \quad \Rightarrow x = 143.1°$

[7]