Edexcel A level Maths Trigonometric functions

Topic Assessment

1. Find all of the angles between 0° and 360° such that
   (i) $\sec x = 2.5$ [2]
   (ii) $\operatorname{cosec} x = -1.5$ [2]

2. Sketch the curve $y = 1 + \sec 2x$ for $0 \le x \le 2\pi$.
   Give the equations of the asymptotes and the coordinates of the turning points. [6]

3. Solve the equation $\sec^2 x + \tan x = 1$ for $0 \le x \le 2\pi$. [5]

4. (i) Solve the equation $\cot x = \sin x$ for $0° \le x \le 360°$. [5]
   (ii) Sketch the graphs of $y = \cot x$ and $y = \sin x$ for $0° \le x \le 360°$ on the same axes, and indicate the roots found in part (i) on your graph. [3]

5. A function is defined by $f(x) = 2\cos^{-1} x - 1$.
   (i) Write down the domain and range of this function. [2]
   (ii) Find the exact value of f(-0.5). [2]
   (iii) Find the inverse function $f^{-1}(x)$. [3]

6. Prove the following identities.
   (i) $(\sec^2 x + \tan^2 x)(\operatorname{cosec}^2 x + \cot^2 x) \equiv 1 + 2\sec^2 x \operatorname{cosec}^2 x$ [5]
   (ii) $\dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x}{1 - \cot x} \equiv \sin x + \cos x$ [5]

Total 40 marks

Edexcel A level Trig functions Assessment solns

Solutions to Topic Assessment

1. (i) $\sec x = 2.5$

$\cos x = 0.4$

$x = 66.4°$ or $293.6°$

[2]

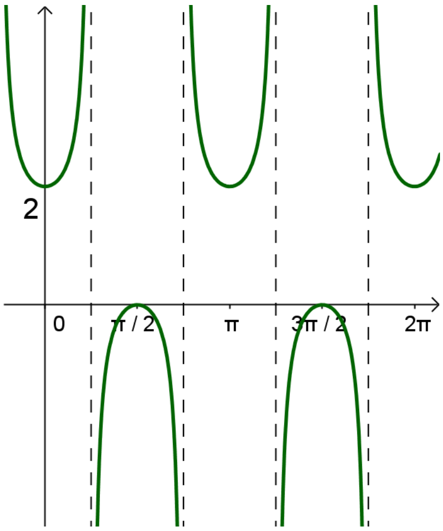
(ii) $\text{cosec}\, x = -1.5$

$\sin x = -\frac{2}{3}$

$x = 221.8°$ or $318.2°$

[2]

2.



[2]

Asymptotes are $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, $x = \frac{7\pi}{4}$ [2]

Turning points are $(0, 2)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, 2)$, $\left(\frac{3\pi}{2}, 0\right)$, $(2\pi, 2)$ [2]

3. $\sec^2 x + \tan x = 1$

$1 + \tan^2 x + \tan x = 1$

$\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ or -1

$\tan x = 0 \quad \Rightarrow x = 0, \pi, 2\pi$

$\tan x = -1 \quad \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

[5]

Edexcel A level Trig functions Assessment solns

4. (i) $\cot x = \sin x$

$$\frac{\cos x}{\sin x} = \sin x$$

$$\cos x = \sin^2 x$$

$$\cos x = 1 - \cos^2 x$$

$$\cos^2 x + \cos x - 1 = 0$$

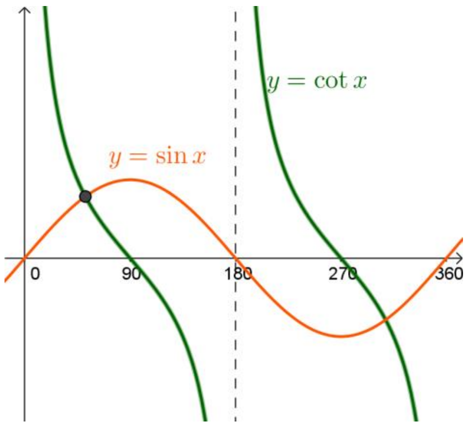
$$\cos x = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\cos x = \frac{-1 - \sqrt{5}}{2} = -1.62 \text{ so no real solutions}$$

$$\cos x = \frac{-1 + \sqrt{5}}{2} \Rightarrow x = 51.8^\circ \text{ or } 308.2^\circ$$

[5]

(ii)



[3]

5. $f(x) = 2\cos^{-1} x - 1$

(i) Domain for $f(x)$ is $-1 \le x \le 1$
Range for $\cos^{-1} x$ is $0 \le x \le \pi$, so range for $f(x)$ is $-1 \le f(x) \le 2\pi - 1$.

[2]

(ii) $f(x) = 2\cos^{-1}(-0.5) - 1$

$$= 2 \times \frac{2\pi}{3} - 1$$

$$= \frac{4\pi}{3} - 1$$

[2]

(iii) $y = 2\cos^{-1} x - 1$

$y + 1 = 2\cos^{-1} x$

$\dfrac{y+1}{2} = \cos^{-1} x$

$x = \cos\left(\dfrac{y+1}{2}\right)$

The inverse function is $f^{-1}(x) = \cos\left(\dfrac{x+1}{2}\right)$

[3]

6. (i) $\text{LHS} \equiv (\sec^2 x + \tan^2 x)(\text{cosec}^2 x + \cot^2 x)$

$\equiv \sec^2 x \,\text{cosec}^2 x + \tan^2 x \,\text{cosec}^2 x + \sec^2 x \cot^2 x + \tan^2 x \cot^2 x$

$\equiv \sec^2 x \,\text{cosec}^2 x + \dfrac{\sin^2 x}{\cos^2 x} \times \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} \times \dfrac{\cos^2 x}{\sin^2 x} + 1$

$\equiv \sec^2 x \,\text{cosec}^2 x + \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x} + 1$

$\equiv \sec^2 x \,\text{cosec}^2 x + \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} + 1$

$\equiv \sec^2 x \,\text{cosec}^2 x + \dfrac{1}{\cos^2 x \sin^2 x} + 1$

$\equiv \sec^2 x \,\text{cosec}^2 x + \sec^2 x \,\text{cosec}^2 x + 1$

$\equiv 1 + 2\sec^2 x \,\text{cosec}^2 x$

$\equiv \text{RHS}$

[5]

(ii) $\text{LHS} \equiv \dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x}{1 - \cot x}$

$\equiv \dfrac{\cos^2 x}{\cos x(1 - \tan x)} + \dfrac{\sin^2 x}{\sin x(1 - \cot x)}$

$\equiv \dfrac{\cos^2 x}{\cos x - \sin x} + \dfrac{\sin^2 x}{\sin x - \cos x}$

$\equiv \dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

$\equiv \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$

$\equiv \sin x + \cos x$

$\equiv \text{RHS}$