Topic assessment

1. Expand $(3-x)^{-4}$ in ascending powers of x up to and including the term in x^3, stating the range for which the expansion is valid. [5]

2. Simplify $\dfrac{4x^2-1}{2x^2+5x-3}$. [3]

3. Write as a single fraction in its simplest form $\dfrac{x}{x-3}-\dfrac{3x}{x^2-9}$. [4]

4. Write $\dfrac{6}{9x^2-1}$ as a sum of two partial fractions. [4]

5. Given that $\dfrac{x+15}{(x-1)(x+3)} \equiv \dfrac{A}{x-1}+\dfrac{B}{x+3}$ find values for A and B. [4]

6. Divide $\dfrac{x^2-3x-4}{x^2-25}$ by $\dfrac{x+1}{x-5}$. [4]

7. Divide x^3-2x^2+3 by $x+3$. [5]

8. $\dfrac{1+x}{1-2x}$ is approximately equal to $1+ax+bx^2$. Find the values of a and b. [6]

9. (i) Write $\dfrac{9}{(1-x)(1+2x)^2}$ as partial fractions. [5]

   (ii) Using your answer to part (i), expand $\dfrac{9}{(1-x)(1+2x)^2}$ up to and including the term in x^2, stating the range of values for which your expansion is valid. [7]

10. (i) Write $f(x)=\dfrac{4}{(x-1)(x+3)}$ as partial fractions. [4]

    (ii) Hence show that $f'(x)=\dfrac{-1}{(x-1)^2}+\dfrac{1}{(x+3)^2}$. [3]

    (iii) Find the x co-ordinate(s) of any turning points on the curve $y=f(x)$. [3]

    (iv) Find $f''(x)$ and hence identify the nature of the turning point(s). [3]

Total 60 marks

Edexcel A level Maths Algebra Assessment solutions

Solutions to Topic Assessment

1. $(3-x)^{-4} = 3^{-4}(1-\tfrac{1}{3}x)^{-4}$

$$= \tfrac{1}{81}\left(1+(-4)(-\tfrac{1}{3}x)+\frac{-4\times-5}{1\times2}(-\tfrac{1}{3}x)^2+\frac{-4\times-5\times-6}{1\times2\times3}(-\tfrac{1}{3}x)^3+\ldots\right)$$

$$= \tfrac{1}{81}\left(1+\tfrac{4}{3}x+\tfrac{10}{9}x^2+\tfrac{20}{27}x^3+\ldots\right)$$

$$= \tfrac{1}{81}+\tfrac{4}{243}x+\tfrac{10}{729}x^2+\tfrac{20}{2187}x^3+\ldots$$

Expansion is valid for $-1 < \tfrac{1}{3}x < 1$

$-3 < x < 3$

[5]

2. $$\frac{4x^2-1}{2x^2+5x-3} = \frac{(2x-1)(2x+1)}{(x+3)(2x-1)} = \frac{2x+1}{x+3} \quad \left(\text{or } 2-\frac{5}{x+3}\right)$$

[3]

3. $$\frac{x}{x-3}-\frac{3x}{x^2-9} \equiv \frac{x}{x-3}-\frac{3x}{(x-3)(x+3)}$$

$$\equiv \frac{x(x+3)}{(x-3)(x+3)}-\frac{3x}{(x-3)(x+3)}$$

$$\equiv \frac{x^2+3x-3x}{(x-3)(x+3)}$$

$$\equiv \frac{x^2}{(x-3)(x+3)}$$

[4]

4. $$\frac{6}{9x^2-1} \equiv \frac{6}{(3x-1)(3x+1)} \equiv \frac{A}{3x-1}+\frac{B}{3x+1}$$

$6 \equiv A(3x+1)+B(3x-1)$

Putting $x=\tfrac{1}{3}$ $\Rightarrow 6=2A$ $\Rightarrow A=3$

Putting $x=-\tfrac{1}{3}$ $\Rightarrow 6=-2B$ $\Rightarrow B=-3$

$$\frac{6}{9x^2-1} \equiv \frac{3}{3x-1}-\frac{3}{3x+1}$$

[4]

5. $$\frac{x+15}{(x-1)(x+3)} \equiv \frac{A}{x-1}+\frac{B}{x+3}$$

$x+15 \equiv A(x+3)+B(x-1)$

Putting $x=1$ $\Rightarrow 16=4A$ $\Rightarrow A=4$

Putting $x=-3$ $\Rightarrow 12=-4B$ $\Rightarrow B=-3$

$$\frac{x+15}{(x-1)(x+3)} \equiv \frac{4}{x-1}-\frac{3}{x+3}$$

[4]

6. $$\frac{x^2-3x-4}{x^2-25} \div \frac{x+1}{x-5} = \frac{(x-4)\cancel{(x+1)}}{\cancel{(x-5)}(x+5)} \times \frac{\cancel{x-5}}{\cancel{x+1}}$$
$$= \frac{x-4}{x+5}$$

[4]

7. $x^3-2x^2+3=(x+3)(x^2-5x+15)-42$
so $(x^3-2x^2+3)\div(x+3)=x^2-5x+15$ remainder -42

[4]

8. $$\frac{1+x}{1-2x}=(1+x)(1-2x)^{-1}$$
$$=(1+x)\left(1+(-1)(-2x)+\frac{-1\times-2}{1\times2}(-2x)^2+\ldots\right)$$
$$=(1+x)\left(1+2x+4x^2+\ldots\right)$$
$$=1+2x+4x^2+x+2x^2+\ldots$$
$$=1+3x+6x^2+\ldots$$
$a=3$, $b=6$

[6]

9. (i) $$\frac{9}{(1-x)(1+2x)^2}\equiv\frac{A}{1-x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$$
$9\equiv A(1+2x)^2+B(1-x)(1+2x)+C(1-x)$
Putting $x=1 \;\Rightarrow 9=9A \;\Rightarrow A=1$
Putting $x=-\frac{1}{2} \;\Rightarrow 9=\frac{3}{2}C \;\Rightarrow C=6$
Equating coefficients of $x^2 \Rightarrow 0=4A-2B \Rightarrow B=2$
$$\frac{9}{(1-x)(1+2x)^2}\equiv\frac{1}{1-x}+\frac{2}{1+2x}+\frac{6}{(1+2x)^2}$$

[5]

(ii) $$\frac{9}{(1-x)(1+2x)^2}=(1-x)^{-1}+2(1+2x)^{-1}+6(1+2x)^{-2}$$
$$(1-x)^{-1}=1+(-1)(-x)+\frac{-1\times-2}{1\times2}(-x)^2+\ldots$$
$$=1+x+x^2+\ldots$$
$$(1+2x)^{-1}=1+(-1)(2x)+\frac{-1\times-2}{1\times2}(2x)^2+\ldots$$
$$=1-2x+4x^2+\ldots$$
$$(1+2x)^{-2}=1+(-2)(2x)+\frac{-2\times-3}{1\times2}(2x)^2+\ldots$$
$$=1-4x+12x^2+\ldots$$

$$\frac{9}{(1-x)(1+2x)^2} = (1+x+x^2)+2(1-2x+4x^2)+6(1-4x+12x^2)+\dots$$
$$= 1+x+x^2+2-4x+8x^2+6-24x+72x^2+\dots$$
$$= 9-27x+81x^2+\dots$$

1st expansion is valid for $-1 < x < 1$

2nd and 3rd expansions are valid for $-1 < 2x < 1 \quad \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

So the expansion is valid for $-\frac{1}{2} < x < \frac{1}{2}$

[8]

10. (i) $f(x) = \dfrac{4}{(x-1)(x+3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+3}$

$4 = A(x+3) + B(x-1)$

Putting $x = 1 \quad \Rightarrow 4 = 4A \quad \Rightarrow A = 1$

Putting $x = -3 \quad \Rightarrow 4 = -4B \quad \Rightarrow B = -1$

$$f(x) = \frac{1}{x-1} - \frac{1}{x+3}$$

[4]

(ii) $f(x) = \dfrac{1}{x-1} - \dfrac{1}{x+3} = (x-1)^{-1} - (x+3)^{-1}$

$$f'(x) = -(x-1)^{-2} + (x+3)^{-2}$$
$$= \frac{-1}{(x-1)^2} + \frac{1}{(x+3)^2}$$

[3]

(iii) At turning points, $f'(x) = 0$

$$\frac{-1}{(x-1)^2} + \frac{1}{(x+3)^2} = 0$$
$$(x+3)^2 = (x-1)^2$$
$$x^2+6x+9 = x^2-2x+1$$
$$8x = -8$$
$$x = -1$$

The only turning point is at $x = -1$.

[3]

(iv) $f'(x) = -(x-1)^{-2} + (x+3)^{-2}$

$$f''(x) = -2 \times -(x-1)^{-3} - 2(x+3)^{-3}$$
$$= \frac{2}{(x-1)^3} - \frac{2}{(x+3)^3}$$

When $x = -1$, $f''(x) = \dfrac{2}{(-2)^3} - \dfrac{2}{(2)^3} = -\dfrac{1}{4} - \dfrac{1}{4} < 0$

so the turning point is a maximum point.

[3]