Topic assessment

1. The functions f, g and h are defined as follows:

$$f(x) = e^x \qquad x \in \mathbb{R}$$
$$g(x) = \sqrt{x} \qquad x \geq 0$$
$$h(x) = 2x + 1 \qquad x \in \mathbb{R}$$

Find each of the following functions, giving the domain and range of each.

(i) fg(x) (ii) gh(x) (iii) hf(x) [9]

(iv) $f^{-1}(x)$ (v) $h^{-1}(x)$ [6]

2. (i) Sketch the graph of $y = |2x + 1|$. [2]

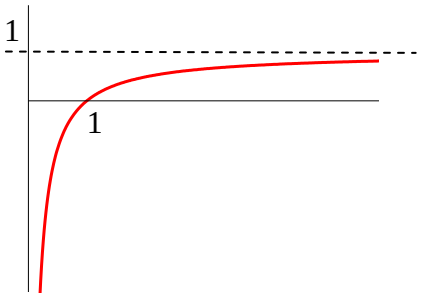
(ii) Hence, or otherwise, solve each of the following equations:

(a) $|2x + 1| = 3 - x$ [3]

(b) $|2x + 1| = 3x - 2$ [3]

3. The diagram below shows the graph $y = f(x)$, where $f(x) = \dfrac{x-1}{x}$ for $x > 0$.

The graph approaches the line $y = 1$ as x becomes very large.



(i) Write down the domain and range of f(x). [2]

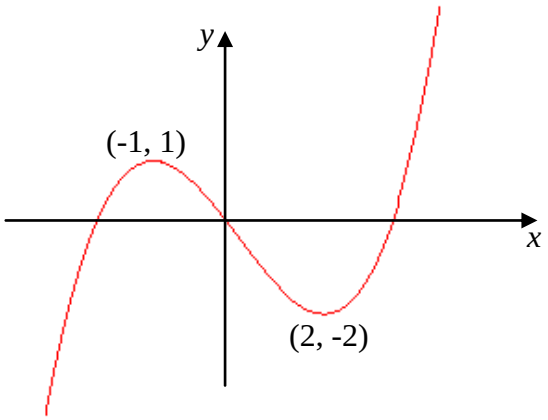
(ii) Find the inverse function $f^{-1}(x)$. [3]

(iii) Write down the domain and range of $f^{-1}(x)$. [2]

(iv) Sketch the graph of $y = f^{-1}(x)$ for the domain you gave in (iii). [2]

(v) What is the relationship between the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$? [1]

4. The graph of a function $y = f(x)$ is shown below. The graph has a local maximum at (–1, 1) and a local minimum at (2, –2).



Sketch the graphs of:

(i) $y = 3\mathrm{f}(2x)$ [3]

(ii) $y = 2\mathrm{f}(x-1)$ [3]

(iii) $y = \mathrm{f}(2x)-1$ [3]

(iv) $y = \mathrm{f}(-x)+1$ [3]

giving the coordinates of the turning points in each case.

5. (i) Solve the inequality $|3x-2| \leq 4$. [3]

(ii) Write the inequality $-2 < x < 7$ in the form $|x-a| < b$. [2]

Total 50 marks

Edexcel A level Maths Functions Assessment solns

Solutions to topic assessment

1. (i) $fg(x) = f(\sqrt{x}) = e^{\sqrt{x}}$

Domain is $x \geq 0$

Range is $fg(x) \geq 1$

[3]

(ii) $gh(x) = g(2x+1) = \sqrt{2x+1}$

Domain is $x \geq -\frac{1}{2}$

Range is $gh(x) \geq 0$.

[3]

Since g(x) is defined as a function, only the positive square root is used, giving this range.

(iii) $hf(x) = h(e^x) = 2e^x + 1$

Domain is $x \in \mathbb{R}$

Range is $hf(x) > 1$

[3]

(iv) $y = e^x$

$\ln y = x$

$f^{-1}(x) = \ln x$

Domain is $x > 0$

Range is $f^{-1}(x) \in \mathbb{R}$

[3]

(v) $y = 2x + 1$

$y - 1 = 2x$

$x = \dfrac{y-1}{2}$

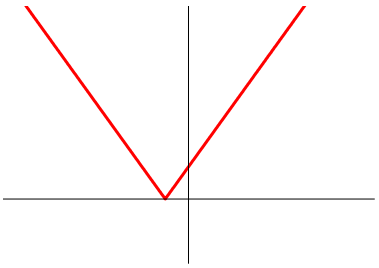
$h^{-1}(x) = \dfrac{x-1}{2}$

Domain is $x \in \mathbb{R}$

Range is $h^{-1}(x) \in \mathbb{R}$

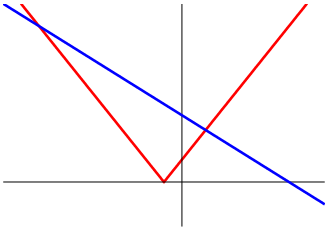
[3]

2. (i) $y = |2x + 1|$



[2]

(ii) (a) $|2x+1| = 3-x$



The graph shows that there are two roots.

$2x+1 = 3-x$ $\quad$ $-(2x+1) = 3-x$

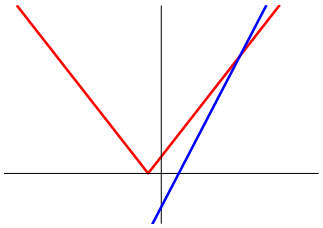
$3x = 2$ $\quad$ $-2x-1 = 3-x$

$x = \frac{2}{3}$ $\quad$ $-4 = x$

So $x = \frac{2}{3}$ and $x = -4$

[3]

(b) $|2x+1| = 3x-2$



The graph shows that there is just one root

$2x+1 = 3x-2$

$3 = x$

The root is $x = 3$.

[3]

3. (i) Domain is $x > 0$

Range is $f(x) < 1$

[2]

(ii) $y = \dfrac{x-1}{x}$

$xy = x-1$

$1 = x-xy$

$1 = x(1-y)$

$x = \dfrac{1}{1-y}$

$f^{-1}(x) = \dfrac{1}{1-x}$

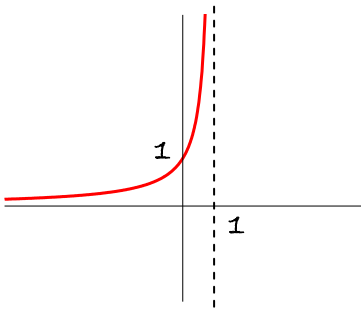
[3]

(iii) Domain is $x < 1$

Range is $f(x) > 0$

[2]

(iv)



[2]

(v) They are reflections of each other in the line $y = x$.

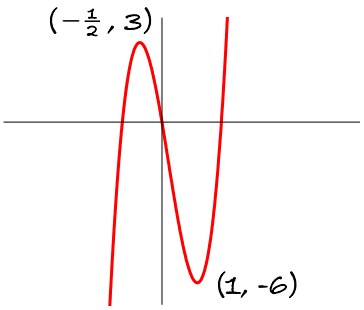
[1]

Edexcel A level Maths Functions Assessment solns

4. (i) $y = 3f(2x)$

The graph is stretched scale factor 3 in the y direction, and scale factor $\frac{1}{2}$ in the x direction.

The turning points are (1, -6) and $(-\frac{1}{2}, 3)$.

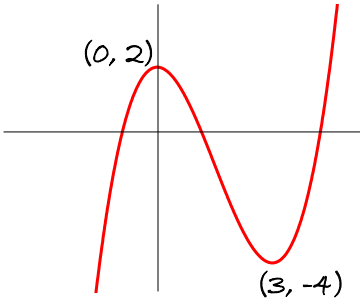


[3]

(ii) $y = 2f(x-1)$

The graph is translated 1 unit horizontally to the right, and stretched scale factor 2 in the y direction.

The turning points are (3, -4) and (0, 2)

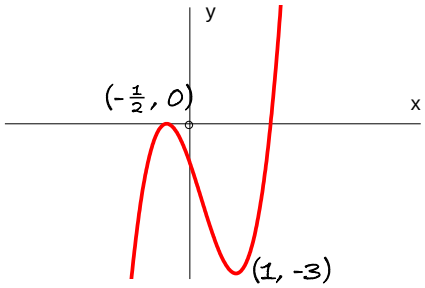


[3]

(iii) $y = f(2x) - 1$

The graph is stretched scale factor $\frac{1}{2}$ parallel to the x axis, and translated 1 unit downwards.

The turning points are (1, -3) and $(-\frac{1}{2}, 0)$

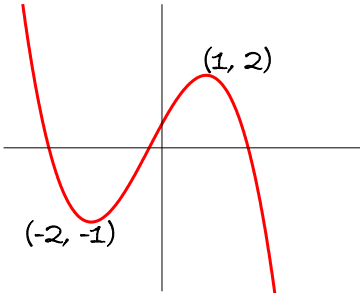


[3]

(iv) $y = f(-x) + 1$

The graph is reflected in the y-axis, and translated 1 unit upwards.

The turning points are (-2, -1) and (1, 2).



[3]

5. (i) $|3x - 2| \leq 4$

$-4 \leq 3x - 2 \leq 4$

$-2 \leq 3x \leq 6$

$-\frac{2}{3} \leq x \leq 2$

[3]

(ii) $-2 < x < 7$

$-2 - 2.5 < x - 2.5 < 7 - 2.5$

$-4.5 < x - 2.5 < 4.5$

$|x - 2.5| < 4.5$

[2]