Section 2: General equations

Solutions to Exercise level 3

1. $y = ut\sin\alpha - \tfrac{1}{2}gt^2$

When $y = 1.2$, $1.2 = ut\sin\alpha - 5t^2$

$$5t^2 - ut\sin\alpha + 1.2 = 0$$

$$t = \frac{u\sin\alpha \pm \sqrt{u^2\sin^2\alpha - 24}}{10}$$

Difference between times = 1 second

$$\Rightarrow \frac{u\sin\alpha + \sqrt{u^2\sin^2\alpha - 24}}{10} - \frac{u\sin\alpha - \sqrt{u^2\sin^2\alpha - 24}}{10} = 1$$

$$\Rightarrow 2\sqrt{u^2\sin^2\alpha - 24} = 10$$

$$\Rightarrow \sqrt{u^2\sin^2\alpha - 24} = 5$$

$$\Rightarrow u^2\sin^2\alpha - 24 = 25$$

$$\Rightarrow u^2\sin^2\alpha = 49$$

$$\Rightarrow u\sin\alpha = 7$$

Distance between horizontal points = 7

$$\Rightarrow ut_2\cos\alpha - ut_1\cos\alpha = 7$$

$$\Rightarrow u(t_2 - t_1)\cos\alpha = 7$$

$$\Rightarrow u\cos\alpha = 7 \quad (\text{since } t_2 - t_1 = 1)$$

Hence $\dfrac{u\sin\alpha}{u\cos\alpha} = \dfrac{7}{7}$

$$\Rightarrow \tan\alpha = 1$$

$$\Rightarrow \alpha = 45°$$

2. (i) Horizontally: $x = ut\cos\theta \quad \Rightarrow t = \dfrac{x}{u\cos\theta}$

Vertically: $y = ut\sin\theta - \tfrac{1}{2}gt^2$

$$= u\left(\frac{x}{u\cos\theta}\right)\sin\theta - \tfrac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$= x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = -\frac{g}{2u^2\cos^2\theta}\left(x^2 - \frac{2u^2\cos^2\theta\tan\theta}{g}x\right)$$

$$= -\frac{g}{2u^2\cos^2\theta}\left[\left(x - \frac{u^2\cos^2\theta\tan\theta}{g}\right)^2 - \frac{u^4\cos^4\theta\tan^2\theta}{g^2}\right]$$

$$= -\frac{g}{2u^2\cos^2\theta}\left(x - \frac{u^2\cos^2\theta\tan\theta}{g}\right)^2 + \frac{u^2\cos^2\theta\tan^2\theta}{2g}$$

$$= -\frac{g}{2u^2\cos^2\theta}\left(x - \frac{u^2\cos^2\theta\tan\theta}{g}\right)^2 + \frac{u^2\sin^2\theta}{2g}$$

So the maximum height is given by $\dfrac{u^2\sin^2\theta}{2g}$.

The maximum height is when $x = \dfrac{u^2\cos^2\theta\tan\theta}{g}$

(ii) $y = x\tan\theta - \dfrac{gx^2}{2u^2}\left(1+\tan^2\theta\right)$

$y_0 = x_0\tan\theta - \dfrac{gx_0^2}{2u^2}\left(1+\tan^2\theta\right)$

$2u^2y_0 = 2u^2x_0\tan\theta - gx_0^2 - gx_0^2\tan^2\theta$

$gx_0^2\tan^2\theta - 2u^2x_0\tan\theta + gx_0^2 + 2u^2y_0 = 0$

(A) Two distinct roots $\Rightarrow$ discriminant > 0

$(-2u^2x_0)^2 - 4\times gx_0^2(gx_0^2 + 2u^2y_0) > 0$

$4u^4x_0^2 - 4gx_0^2(gx_0^2 + 2u^2y_0) > 0$

$u^4 - g^2x_0^2 - 2u^2gy_0 > 0$

(B) Two equal roots $\Rightarrow$ discrimant $= 0$

$u^4 - g^2x_0^2 - 2u^2gy_0 = 0$

(C) No roots $\Rightarrow$ discrimant < 0

$u^4 - g^2x_0^2 - 2u^2gy_0 < 0$

If the point is inaccessible, there are no roots, so

$u^4 - g^2x_0^2 - 2u^2gy_0 < 0$

$2u^2gy_0 > u^4 - g^2x_0^2$

$y_0 > \dfrac{u^2}{2g} - \dfrac{gx_0^2}{2u^2}$

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3. Vertically: $v^2 = u^2 + 2as$

$$0 = (u\sin\theta)^2 - 2gh$$

$$h = \frac{u^2\sin^2\theta}{2g}$$

Horizontally: $x = ut\cos\theta$, so at P, $d = ut\cos\theta \Rightarrow t = \frac{d}{u\cos\theta}$

Vertically: $y = ut\sin\theta - \frac{1}{2}gt^2$

so at P, $\frac{1}{2}d = u\left(\frac{d}{u\cos\theta}\right)\sin\theta - \frac{1}{2}g\left(\frac{d}{u\cos\theta}\right)^2$

$$d = 2d\tan\theta - \frac{gd^2}{u^2\cos^2\theta}$$

$$\frac{gd}{u^2\cos^2\theta} = 2\tan\theta - 1$$

Maximum height $< \frac{9d}{10}$

$$\Rightarrow \frac{u^2\sin^2\theta}{2g} < \frac{9d}{10}$$

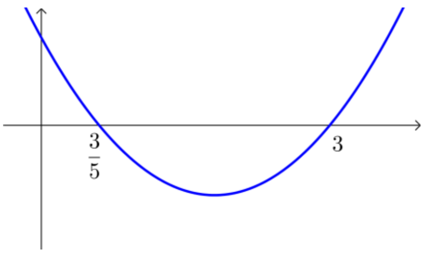
$$\Rightarrow \frac{u^2\tan^2\theta}{2g} < \frac{9d}{10\cos^2\theta}$$

$$\Rightarrow \tan^2\theta < \frac{9gd}{5u^2\cos^2\theta}$$

$$\Rightarrow \tan^2\theta < \frac{9}{5}(2\tan\theta - 1)$$

$$\Rightarrow 5\tan^2\theta - 18\tan\theta + 9 < 0$$

$$\Rightarrow (\tan\theta - 3)(5\tan\theta - 3) < 0$$



so $\frac{3}{5} < \tan\theta < 3$

$$\Rightarrow \arctan\frac{3}{5} < \theta < \arctan 3$$