Section 2: General equations

Solutions to Exercise level 2

1. Horizontally: $x = ut$

$t = \frac{x}{u}$

Vertically: $y = -\frac{1}{2}gt^2$

$y = -\frac{g}{2}\left(\frac{x}{u}\right)^2$

$y = -\frac{gx^2}{2u^2}$

(i) If it just clears the fence, $y = -2.5$ when $x = 8$.

$-2.5 = -\frac{9.8 \times 8^2}{2u^2}$

$u^2 = 125.44$

$u = 11.2$

The initial speed is 11.2 ms^{-1}.

(ii) When ball lands, $y = -4.9$

$-4.9 = -\frac{9.8x^2}{2 \times 11.2^2}$

$x^2 = \frac{4.9 \times 2 \times 11.2^2}{9.8}$

$x = 11.2$

It lands 11.2 m horizontally from its starting point so it lands 3.2 m beyond the fence.

2. Vertically: $y = ut\sin\theta - \frac{1}{2}gt^2$

When shell hits ground, $0 = t(210\sin\theta - 4.9t)$

$t = \frac{210\sin\theta}{4.9}$

Horizontally: $x = ut\cos\theta$

$= 210 \times \frac{210\sin\theta}{4.9} \times \cos\theta$

$= 9000\sin\theta\cos\theta$

$= 4500\sin 2\theta$

(i) At maximum range, angle = 45°.

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$\text{Range} = 4500 \times \sin 90°$

$= 4500$

The maximum range is 4.5 km.

(ii) $3600 = 4500 \sin 2\theta$

$\sin 2\theta = 0.8$

$2\theta = 53.13°$ or $126.87°$

$\theta = 26.6°$ or $63.4°$

3. (i) Horizontally: $x = ut\cos\theta$

$22 = 2u\cos\theta$

$u\cos\theta = 11$

Vertically: $y = ut\sin\theta - \frac{1}{2}gt^2$

$2.4 = 2u\sin\theta - \frac{1}{2} \times 9.8 \times 4$

$u\sin\theta = 11$

Dividing: $\tan\theta = 1 \quad \Rightarrow \theta = 45°$

$u = \dfrac{11}{\sin 45°} = 11\sqrt{2} = 15.6 \text{ ms}^{-1}$ (3 s.f.)

It was kicked at 15.6 ms^{-1} at 45° to the horizontal.

(ii) At maximum height: $v_y^2 = u_y^2 - 2gs$

$0 = (11\sqrt{2}\sin 45°)^2 - 2 \times 9.8s$

$s = \dfrac{121}{19.6} = 6.17$ m (3 s.f.)

(iii) When ball lands: $y = ut\sin\theta - \frac{1}{2}gt^2$

$0 = t(11 - 4.9t)$

$t = \dfrac{11}{4.9}$

$x = ut\cos\theta = 11t = 11 \times \dfrac{11}{4.9} = 24.7$

The ball lands 24.7 m from O (3 s.f.)

4. (i) At greatest height: $v_y^2 = u_y^2 - 2gs$

$0 = (25\sin 50°)^2 - 2 \times 9.8s$

$s = \dfrac{(25\sin 50°)^2}{19.6} = 18.7$ m (3 s.f.)

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(ii) When $x = 50$, $50 = 25t\cos 50°$

$$t = \frac{2}{\cos 50°}$$

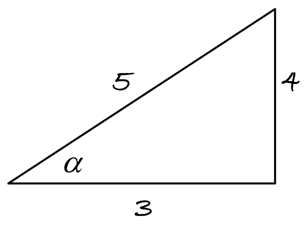
At this time, $y = 25t\sin 50° - \frac{1}{2} \times 9.8t^2$

$$= 25\sin 50° \times \frac{2}{\cos 50°} - 4.9\left(\frac{2}{\cos 50°}\right)^2$$

$$= 12.15$$

The ball will clear the pavilion.

5. (i)



$$y = 60t\sin\alpha - \tfrac{1}{2}gt^2$$

$$99 = 60t \times \tfrac{4}{5} - 5t^2$$

$$5t^2 - 48t + 99 = 0$$

$$(5t - 33)(t - 3) = 0$$

$$t = 3 \quad \text{or} \quad t = 6.6$$

The particle is at a height of 99 m at 3 seconds and at 6.6 seconds.

(ii) $x = 60t\cos\alpha = 60t \times \frac{3}{5} = 36t$

When $t = 3$, $x = 36 \times 3 = 108$

When $t = 6.6$, $x = 36 \times 6.6 = 237.6$

When the particle is at a height of 99 m, it is at a horizontal distance of 108 m or 237.6 m from O.