Section 1: Introduction

Exercise level 3 (Extension)

1. Let $OA = OB = d$

Area $OAC = \frac{1}{2} \times OC \times AC$

$= \frac{1}{2} \times d\cos\theta \times d\sin\theta$

$= \frac{1}{2} d^2 \sin\theta\cos\theta$

so area of $OAB = 2 \times \frac{1}{2} d^2 \sin\theta\cos\theta = d^2 \sin\theta\cos\theta$

Using formula for area of triangle:

Area $OAB = \frac{1}{2} \times OA \times OB \sin 2\theta$

$= \frac{1}{2} d^2 \sin 2\theta$

So $\frac{1}{2} d^2 \sin 2\theta = d^2 \sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = 2\sin\theta\cos\theta$

Vertically for complete flight of particle:

$y = ut\sin\alpha - \frac{1}{2} gt^2$

$0 = ut\sin\alpha - \frac{1}{2} gt^2$

$0 = t(u\sin\alpha - \frac{1}{2} gt)$

so time of flight $= \dfrac{2u\sin\alpha}{g}$

Range $= ut\cos\alpha$

$= u\cos\alpha \times \dfrac{2u\sin\alpha}{g}$

$= \dfrac{2u^2 \sin\alpha\cos\alpha}{g}$

$= \dfrac{u^2 \sin 2\alpha}{g}$

So maximum range is when $\sin 2\alpha = 1$

$\Rightarrow 2\alpha = 90°$

$\Rightarrow \alpha = 45°$

2. (i) From question 1, range $= \dfrac{u^2 \sin 2\alpha}{g}$

$= \dfrac{40^2 \sin 60°}{10}$

$= 80\sqrt{3}$

Edexcel A level Maths Projectiles 1 Exercise solutions

(ii) $\dfrac{u^2 \sin 2\alpha}{g} = 80\sqrt{3}$

$\dfrac{40^2 \sin 2\alpha}{10} = 80\sqrt{3}$

$\sin 2\alpha = \frac{1}{2}\sqrt{3}$

$2\alpha = 60° \text{ or } 120°$

$\alpha = 30° \text{ or } 60°$

so the other angle is 60°.

(iii) $R = \dfrac{u^2 \sin 2\alpha}{g}$

so for the second angle $\dfrac{u^2 \sin 2\phi}{g} = \dfrac{u^2 \sin 2\theta}{g}$

$\Rightarrow \sin 2\phi = \sin 2\theta$

$\Rightarrow 2\phi = 180° - 2\theta$

$\Rightarrow \phi = 90° - \theta$

(iv) From question 1, time of flight $= \dfrac{2u \sin \alpha}{g} = \dfrac{2u \sin \alpha}{5}$

$\dfrac{2u \sin \phi}{5} = 2 \times \dfrac{2u \sin \theta}{5}$

$\Rightarrow \sin \phi = 2 \sin \theta$

But $\phi = 90° - \theta$, so $\sin(90° - \theta) = 2 \sin \theta$

$\cos \theta = 2 \sin \theta$

$\tan \theta = \frac{1}{2}$

So $\theta = \tan^{-1} \frac{1}{2} = 26.6°$ and $\phi = 63.4°$

For the maximum height, $v^2 = u^2 + 2as$

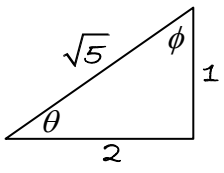
$0 = u^2 \sin^2 \alpha - 2gs$

$s = \dfrac{u^2 \sin^2 \alpha}{20}$

so ratio of maximum heights $= \dfrac{\sin^2 \phi}{\sin^2 \theta}$

$= \dfrac{\left(2/\sqrt{5}\right)^2}{\left(1/\sqrt{5}\right)^2}$

$= 4$



Edexcel A level Maths Projectiles 1 Exercise solutions

3. (i) $x = 4t\cos 75°$

$y = 4t\sin 75° - \frac{1}{2} \times 9.8t^2$

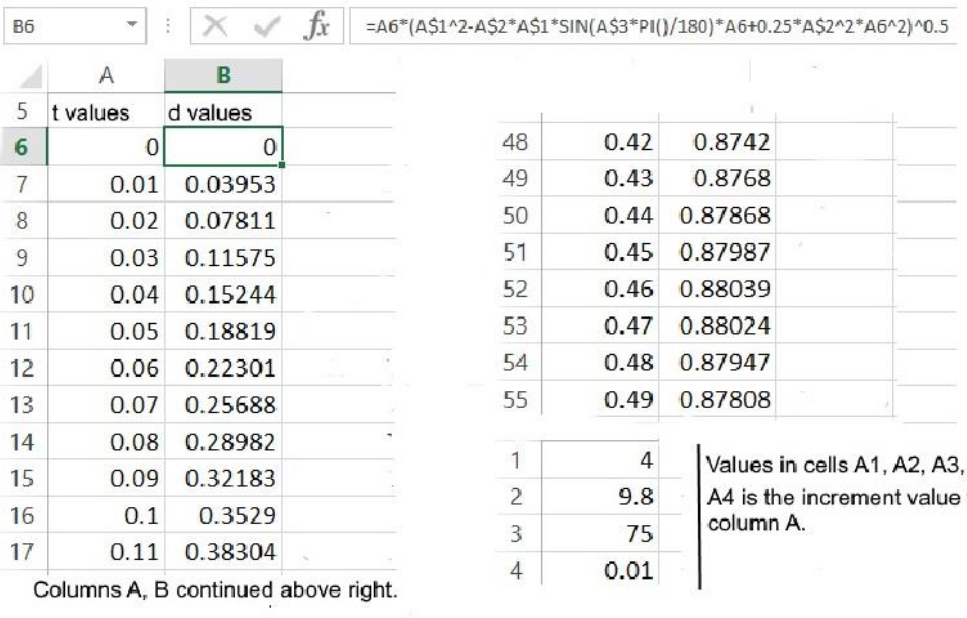
$x^2 + y^2 = 16t^2\cos^2 75° + (4t\sin 75° - 4.9t^2)$

$= 16t^2\cos^2 75° + 16t^2\sin^2 75° - 39.2t^3\sin 75° + 24.01t^4$

$= 16t^2 - 39.2t^3\sin 75° + 24.01t^4$

so distance $d = \sqrt{16t^2 - 39.2t^3\sin 75° + 24.01t^4}$

(ii)



B6 =A6*(A1^2-A2*A$1*SIN(A$3*PI()/180)*A6+0.25*A$2^2*A6^2)^0.5

	A	B
5	t values	d values
6	0	0
7	0.01	0.03953
8	0.02	0.07811
9	0.03	0.11575
10	0.04	0.15244
11	0.05	0.18819
12	0.06	0.22301
13	0.07	0.25688
14	0.08	0.28982
15	0.09	0.32183
16	0.1	0.3529
17	0.11	0.38304

Columns A, B continued above right.

48	0.42	0.8742
49	0.43	0.8768
50	0.44	0.87868
51	0.45	0.87987
52	0.46	0.88039
53	0.47	0.88024
54	0.48	0.87947
55	0.49	0.87808

1	4
2	9.8
3	75
4	0.01

Values in cells A1, A2, A3, A4
A4 is the increment value for column A.

The spreadsheet shows that d is just over 0.88 when t = 0.46 and 0.47. So it just grazes the envelope then.