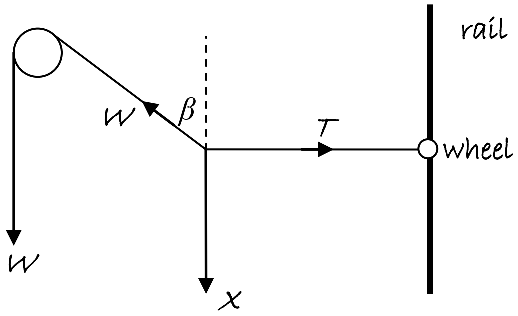
Section 1: Resolving forces

Solutions to Exercise level 3

1.



(i) The tension in the two parts of the first string must be the same, so the tension in the inclined string section is W.

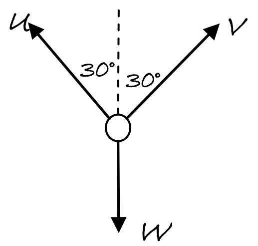
(ii) Considering the forces on the wheel, if the string were not horizontal then there would need to be a vertical force to balance it. Since the wheel is light and the rail is smooth, there is no vertical force.

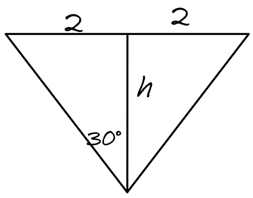
(iii) Resolving vertically: $W\cos\beta = X$

$\beta \neq 0$ so $\cos\beta \neq 1$, so $W \neq X$.

(iv) From (iii) $W = \dfrac{X}{\cos\beta}$

2. (i)





By symmetry $U = V$

Resolving vertically: $2U\cos 30° = W$

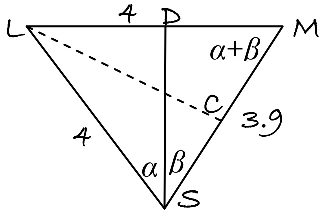
$$U = V = \frac{W}{\sqrt{3}}$$

$\tan 30° = \frac{2}{h}$

$\frac{1}{\sqrt{3}} = \frac{2}{h}$

$h = 2\sqrt{3}$

(ii) Original lengths of LS and SM are both 4 m, since triangle is equilateral.
In new configuration, LMS is isosceles.

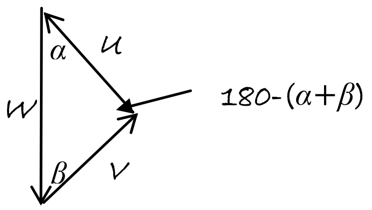


From triangle LSC, $\cos(\alpha + \beta) = \frac{1.95}{4} \Rightarrow \alpha + \beta = 60.824°$

From triangle SDM, $\alpha + 2\beta = 90°$

$60.824° - \beta + 2\beta = 90°$

$\beta = 29.18°,\ \alpha = 31.64°$



$\frac{U}{\sin\beta} = \frac{W}{\sin(180 - (\alpha + \beta))}$

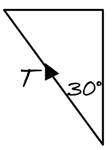
$U = \frac{W\sin\beta}{\sin(\alpha+\beta)} = \frac{W\sin 29.17...}{\sin 60.82...} = 0.558W$

$\frac{V}{\sin\alpha} = \frac{W}{\sin(180 - (\alpha + \beta))}$

$V = \frac{W\sin\alpha}{\sin(\alpha+\beta)} = \frac{W\sin 31.64...}{\sin 60.82...} = 0.601W$

So $U = 0.558W$ (3 s.f.) and $V = 0.601W$ (3 s.f.)

3. (i) For each wire,
vertical component of tension $= T\cos 30°$



Resolving vertically: $4T\cos 30° = 40$

$T = \frac{20}{\sqrt{3}} = 11.54...$

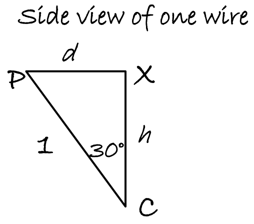
The tension in each wire is 11.5 N (3 s.f.)

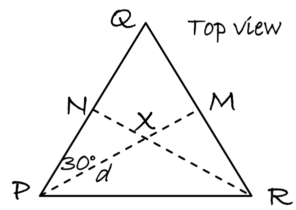
(ii) Resolving vertically: $3T\cos 30° = 40$

$$T = \frac{80}{3\sqrt{3}} = 15.39...$$

The tension in each wire is 15.4 N (3 s.f.)

(iii) The original suspension points are P, Q and R. C is the chandelier and X is the point on the ceiling above the chandelier.
In original configuration:





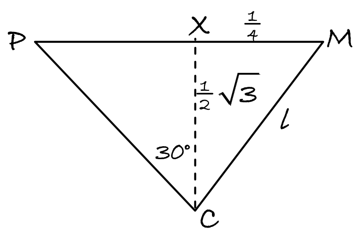
$h = 1 \times \cos 30° = \frac{1}{2}\sqrt{3}$

$d = 1 \times \sin 30° = \frac{1}{2}$

$NX = d\sin 30° = \frac{1}{4}$

By symmetry $MX = \frac{1}{4}$

In new configuration, the wires at Q and R are replaced by a wire from M



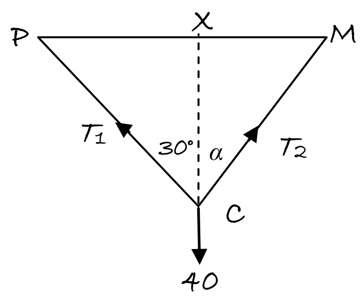
$l^2 = \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\sqrt{3}\right)^2 = \frac{1}{16} + \frac{3}{4} = \frac{13}{16}$

$l = \frac{1}{4}\sqrt{13}$

$l = 0.901...$

The length of the wire is 0.901 m (3 s.f.)

(iv)



Resolving horizontally: $T_2 \sin\alpha = T_1 \sin 30°$

$$T_1 = \frac{2}{\sqrt{13}} T_2$$

Resolving vertically: $T_2 \cos\alpha + T_1 \cos 30° = 40$

$$\frac{2\sqrt{3}}{\sqrt{13}} T_2 + \frac{2}{\sqrt{13}} T_2 \times \frac{\sqrt{3}}{2} = 40$$

$$\frac{3\sqrt{3}}{\sqrt{13}} T_2 = 40$$

$$T_2 = \frac{40\sqrt{13}}{3\sqrt{3}} = 27.55...$$

Tension in wire = 27.6 N (3 s.f.)

(v) From above $T_1 = \frac{2}{\sqrt{13}} T_2 = \frac{2}{\sqrt{13}} \times \frac{40\sqrt{13}}{3\sqrt{3}} = \frac{80}{3\sqrt{3}} = 15.39...$

Tension in other wire = 15.4 N (3 s.f.)

