Section 1: Motion in two dimensions

Solutions to Exercise level 2

1. $\underset{\sim}{r} = 2t^3\underset{\sim}{i} + 3t^2\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = 6t^2\underset{\sim}{i} + 6t\underset{\sim}{j}$

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = 12t\underset{\sim}{i} + 6\underset{\sim}{j}$

When t = 3, $\underset{\sim}{v} = 6\times 3^2\underset{\sim}{i} + 6\times 3\underset{\sim}{j} = 54\underset{\sim}{i} + 18\underset{\sim}{j}$

Magnitude of velocity $= \sqrt{54^2 + 18^2} = 56.9$ ms^{-1}.

When t = 3, $\underset{\sim}{a} = 12\times 3\underset{\sim}{i} + 6\underset{\sim}{j} = 36\underset{\sim}{i} + 6\underset{\sim}{j}$

Magnitude of acceleration $= \sqrt{36^2 + 6^2} = 36.5$ ms^{-2}.

2. $\underset{\sim}{F} = m\underset{\sim}{a}$

$4t\underset{\sim}{i} + 6\underset{\sim}{j} = 2\underset{\sim}{a}$

$\underset{\sim}{a} = 2t\underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{v} = \int \underset{\sim}{a}\,dt = \int\left(2t\underset{\sim}{i} + 3\underset{\sim}{j}\right)dt = t^2\underset{\sim}{i} + 3t\underset{\sim}{j} + \underset{\sim}{c}$

When t = 0, $\underset{\sim}{v} = 5\underset{\sim}{j}$ so $\underset{\sim}{c} = 5\underset{\sim}{j}$

$\underset{\sim}{v} = t^2\underset{\sim}{i} + 3t\underset{\sim}{j} + 5\underset{\sim}{j} = t^2\underset{\sim}{i} + (3t+5)\underset{\sim}{j}$

When t = 3, $\underset{\sim}{v} = 3^2\underset{\sim}{i} + (3\times 3 + 5)\underset{\sim}{j}$

$= 9\underset{\sim}{i} + 14\underset{\sim}{j}$

$\underset{\sim}{r} = \int \underset{\sim}{v}\,dt = \int\left(t^2\underset{\sim}{i} + (3t+5)\underset{\sim}{j}\right)dt = \tfrac{1}{3}t^3\underset{\sim}{i} + (\tfrac{3}{2}t^2 + 5t)\underset{\sim}{j} + \underset{\sim}{d}$

When t = 0, $\underset{\sim}{r} = 0$ so $\underset{\sim}{d} = 0$

$\underset{\sim}{r} = \tfrac{1}{3}t^3\underset{\sim}{i} + (\tfrac{3}{2}t^2 + 5t)\underset{\sim}{j}$

When t = 3, $\underset{\sim}{r} = \tfrac{1}{3}\times 3^3\underset{\sim}{i} + (\tfrac{3}{2}\times 3^2 + 5\times 3)\underset{\sim}{j}$

$= 9\underset{\sim}{i} + \tfrac{57}{2}\underset{\sim}{j}$

3. $\underset{\sim}{r} = 6t\underset{\sim}{i} - 4t^2\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = 6\underset{\sim}{i} - 8t\underset{\sim}{j}$

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = -8\underset{\sim}{j}$

$\underset{\sim}{F} = m\underset{\sim}{a} = 4 \times -8\underset{\sim}{j} = -32\underset{\sim}{j}$

The magnitude of F is 32.

4. (i) $\underset{\sim}{r} = (2t-1)\underset{\sim}{i} - t^2\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = 2\underset{\sim}{i} - 2t\underset{\sim}{j}$

(ii) When t = 0, $\underset{\sim}{v} = 2\underset{\sim}{i}$

so the initial direction of motion is in the positive $\underset{\sim}{i}$ direction.

(iii) $\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = -2\underset{\sim}{j}$ which is not dependent on time, so the acceleration is constant.

(iv) The acceleration acts only in the $\underset{\sim}{j}$ direction, so the component of the velocity in the $\underset{\sim}{i}$ direction is constant. Since the initial velocity is $2\underset{\sim}{i}$, the component of the velocity in the $\underset{\sim}{i}$ direction can never be zero, and so the motion can never be in the $\underset{\sim}{j}$ direction.

(v) $x = 2t - 1 \quad \Rightarrow t = \frac{1}{2}(x+1)$

$y = -t^2 = -\frac{1}{4}(x+1)^2$

The equation of the path is $4y + (x+1)^2 = 0$

5. (i) Resultant force $= \underset{\sim}{F} + \underset{\sim}{T} = 12\underset{\sim}{i} - 2\underset{\sim}{j} + 10\underset{\sim}{i} + 12\underset{\sim}{j}$

$= 22\underset{\sim}{i} + 10\underset{\sim}{j}$

(ii) $\underset{\sim}{F} = m\underset{\sim}{a}$

$22\underset{\sim}{i} + 10\underset{\sim}{j} = 20\underset{\sim}{a}$

$\underset{\sim}{a} = 1.1\underset{\sim}{i} + 0.5\underset{\sim}{j}$

(iii) $\underset{\sim}{u} = 0, \quad \underset{\sim}{a} = 1.1\underset{\sim}{i} + 0.5\underset{\sim}{j}$

Using constant acceleration equation:

Displacement $\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2$

$= 0 + \frac{1}{2}(1.1\underset{\sim}{i} + 0.5\underset{\sim}{j})t^2 = 0.55t^2\underset{\sim}{i} + 0.25t^2\underset{\sim}{j}$

Initial position is $3\underset{\sim}{j}$,

so position vector is $\underset{\sim}{r} = 3\underset{\sim}{j} + 0.55t^2\underset{\sim}{i} + 0.25t^2\underset{\sim}{j}$

$= 0.55t^2\underset{\sim}{i} + \left(3 + 0.25t^2\right)\underset{\sim}{j}$

6. $\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dx} = (2t-4)\underset{\sim}{i} + (3t^2 + 2ft)\underset{\sim}{j}$

Since the particle comes to instantaneous rest, there is a value for t for which

$$2t - 4 = 0 \text{ and } 3t^2 + 2ft = 0$$

$2t - 4 = 0 \Rightarrow t = 2$

When $t = 2$, $3t^2 + 2ft = 0 \Rightarrow 12 + 4f = 0 \Rightarrow f = -3$

7. $\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dx} = 2t\underset{\sim}{i} + 3\underset{\sim}{j}$

When $t = 3$, $\underset{\sim}{a} = 6\underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{s} = \int \underset{\sim}{v}\, dt = \int \left(t^2\underset{\sim}{i} + 3t\underset{\sim}{j}\right) dt$

$= \frac{1}{3}t^3\underset{\sim}{i} + \frac{3}{2}t^2\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 0$, $\underset{\sim}{s} = 18\underset{\sim}{i} - 24\underset{\sim}{j} \Rightarrow \underset{\sim}{c} = 18\underset{\sim}{i} - 24\underset{\sim}{j}$

$\underset{\sim}{s} = \frac{1}{3}t^3\underset{\sim}{i} + \frac{3}{2}t^2\underset{\sim}{j} + 18\underset{\sim}{i} - 24\underset{\sim}{j}$

$= (\frac{1}{3}t^3 + 18)\underset{\sim}{i} + (\frac{3}{2}t^2 - 24)\underset{\sim}{j}$

When $t = 3$, $\underset{\sim}{s} = (9 + 18)\underset{\sim}{i} + (\frac{27}{2} - 24)\underset{\sim}{j}$

$= 27\underset{\sim}{i} - 10.5\underset{\sim}{j}$