

## Section 1: Motion in two dimensions

### Solutions to Exercise level 1

$$1. \quad (i) \quad \underline{v} = \frac{d}{dt} \begin{pmatrix} 3t+1 \\ 2-t \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$(ii) \quad \underline{a} = \frac{d}{dt} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \underline{0}$$

$$2. \quad (i) \quad \underline{v} = \int \left( (3t+2)\underline{i} + (2t^2-5)\underline{j} \right) dt$$

$$= \left( \frac{3}{2}t^2 + 2t \right)\underline{i} + \left( \frac{2}{3}t^3 - 5t \right)\underline{j} + \underline{c}$$

$$\text{When } t=0, \underline{v} = \underline{0} \Rightarrow \underline{c} = \underline{0}$$

$$\underline{v} = \left( \frac{3}{2}t^2 + 2t \right)\underline{i} + \left( \frac{2}{3}t^3 - 5t \right)\underline{j}$$

$$(ii) \quad \underline{s} = \int \left( \left( \frac{3}{2}t^2 + 2t \right)\underline{i} + \left( \frac{2}{3}t^3 - 5t \right)\underline{j} \right) dt$$

$$= \left( \frac{1}{2}t^3 + t^2 \right)\underline{i} + \left( \frac{1}{6}t^4 - \frac{5}{2}t^2 \right)\underline{j} + \underline{d}$$

$$\text{When } t=0, \underline{s} = \underline{0} \Rightarrow \underline{d} = \underline{0}$$

$$\underline{s} = \left( \frac{1}{2}t^3 + t^2 \right)\underline{i} + \left( \frac{1}{6}t^4 - \frac{5}{2}t^2 \right)\underline{j}$$

$$3. \quad (i) \quad \underline{s} = \int \left( 2t\underline{i} + (3t^2 - 4t)\underline{j} \right) dt$$

$$= t^2\underline{i} + (t^3 - 2t^2)\underline{j} + \underline{c}$$

$$\text{When } t=2, \underline{s} = 7\underline{i} + 4\underline{j} \Rightarrow 7\underline{i} + 4\underline{j} = 4\underline{i} + \underline{0} + \underline{c}$$

$$\Rightarrow \underline{c} = 3\underline{i} + 4\underline{j}$$

$$\underline{s} = t^2\underline{i} + (t^3 - 2t^2)\underline{j} + 3\underline{i} + 4\underline{j}$$

$$= (t^2 + 3)\underline{i} + (t^3 - 2t^2 + 4)\underline{j}$$

$$(ii) \quad \text{When } t=5, \underline{s} = (5^2 + 3)\underline{i} + (5^3 - 2 \times 5^2 + 4)\underline{j}$$

$$= 28\underline{i} + 79\underline{j}$$

$$4. \quad (i) \quad \underline{u} = 15\underline{i} + 20\underline{j}$$

$$\underline{a} = -10\underline{j}$$

Using the constant acceleration equations:

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$$\begin{aligned}\text{Velocity vector } \underline{v} &= \underline{u} + \underline{a}t = 15\underline{i} + 20\underline{j} - 10\underline{j} \times t \\ &= 15\underline{i} + (20 - 10t)\underline{j}\end{aligned}$$

$$\begin{aligned}\text{Position vector } \underline{r} &= \underline{u}t + \frac{1}{2}\underline{a}t^2 = (15\underline{i} + 20\underline{j})t + \frac{1}{2} \times -10\underline{j} \times t^2 \\ &= 15t\underline{i} + (20t - 5t^2)\underline{j}\end{aligned}$$

(ii) When the ball is moving in a direction of  $45^\circ$ , the components of the velocity in the  $\underline{i}$  and  $\underline{j}$  directions are equal.

$$15 = 20 - 10t$$

$$10t = 5$$

$$t = 0.5$$

(iii) When the ball returns to the ground, the  $\underline{j}$  component of the position vector is zero.

$$20t - 5t^2 = 0$$

$$t(20 - 5t) = 0$$

$$t = 0 \text{ or } 4$$

It returns to the ground after 4 seconds.

5. (i)  $\underline{u} = 12\underline{i}$

$$\underline{a} = -10\underline{j}$$

Using the constant acceleration equations:

$$\begin{aligned}\text{Velocity } \underline{v} &= \underline{u} + \underline{a}t = 12\underline{i} - 10\underline{j} \times t \\ &= 12\underline{i} - 10t\underline{j}\end{aligned}$$

$$\begin{aligned}\text{Displacement } \underline{s} &= \underline{u}t + \frac{1}{2}\underline{a}t^2 = 12t\underline{i} + \frac{1}{2} \times -10\underline{j} \times t^2 \\ &= 12t\underline{i} - 5t^2\underline{j}\end{aligned}$$

(ii) When the stone lands, the  $\underline{j}$  component of the displacement is  $-10$ .

$$-10 = -5t^2$$

$$t^2 = 2$$

$$t = \sqrt{2}$$

The  $\underline{i}$  component of the displacement at this time is  $12\sqrt{2}$ .

The stone lands  $12\sqrt{2}$  m from the base of the tower.

6.  $\underline{r} = 8t^3\underline{i} + t^4\underline{j}$

$$\underline{v} = \frac{d\underline{r}}{dt} = 24t^2\underline{i} + 4t^3\underline{j}$$

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$$\begin{aligned}\text{When } t = 2, \underline{v} &= 24 \times 2^2 \underline{i} + 4 \times 2^3 \underline{j} \\ &= 96 \underline{i} + 32 \underline{j}\end{aligned}$$

$$\underline{a} = \frac{d\underline{v}}{dt} = 48t \underline{i} + 12t^2 \underline{j}$$

$$\begin{aligned}\text{When } t = 2, \underline{a} &= 48 \times 2 \underline{i} + 12 \times 2^2 \underline{j} \\ &= 96 \underline{i} + 48 \underline{j}\end{aligned}$$

$$7. \underline{v} = 2t \underline{i} + 3t^2 \underline{j}$$

$$\begin{aligned}\underline{s} &= \int \underline{v} dt = \int (2t \underline{i} + 3t^2 \underline{j}) dt \\ &= t^2 \underline{i} + t^3 \underline{j} + \underline{c}\end{aligned}$$

$$\text{When } t = 0, \underline{s} = \underline{i} + 4 \underline{j} \Rightarrow \underline{c} = \underline{i} + 4 \underline{j}$$

$$\begin{aligned}\underline{s} &= t^2 \underline{i} + t^3 \underline{j} + \underline{i} + 4 \underline{j} \\ &= (t^2 + 1) \underline{i} + (t^3 + 4) \underline{j}\end{aligned}$$