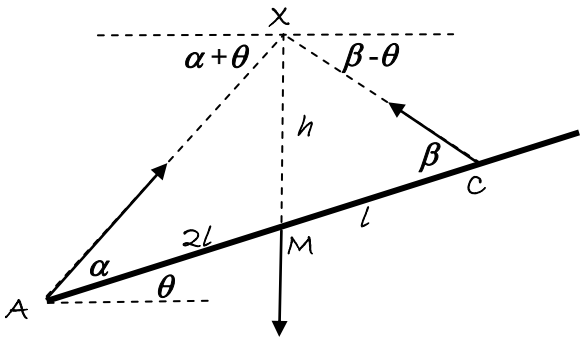
Section 2: Moments of forces at an angle

Solutions to Exercise level 3

1. Since there are three forces acting, they must all act through the same point.



Sine rule for triangle CMX: $\dfrac{\sin\beta}{x}=\dfrac{\sin(90-(\beta-\theta))}{l}$

Sine rule for triangle AMX: $\dfrac{\sin\alpha}{x}=\dfrac{\sin(90-(\alpha+\theta))}{2l}$

$$\frac{\sin\beta}{\sin\alpha}=\frac{2\sin(90-(\beta-\theta))}{\sin(90-(\alpha+\theta))}$$

$$\frac{\sin\beta}{\sin\alpha}=\frac{2\cos(\beta-\theta)}{\cos(\alpha+\theta)}$$

Dividing: $\sin\beta\cos(\alpha+\theta)=2\sin\alpha\cos(\beta-\theta)$

$$\sin\beta\cos\alpha\cos\theta-\sin\beta\sin\alpha\sin\theta=2\sin\alpha\cos\beta\cos\theta+2\sin\alpha\sin\beta\sin\theta$$

$$\sin\beta\cos\alpha\cos\theta=2\sin\alpha\cos\beta\cos\theta+3\sin\alpha\sin\beta\sin\theta$$

Dividing through by $\sin\alpha\sin\beta\cos\theta$:

$$\cot\alpha=2\cot\beta+3\tan\theta$$

$$\cot\alpha=2\cot 45+3\tan 30=2+\sqrt{3}$$

$$\tan\alpha=\frac{1}{2+\sqrt{3}}=\frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}=2-\sqrt{3}$$

Edexcel A level Maths Moments 2 Exercise solutions

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3} - 1)^2}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

$$= \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

so $\tan \alpha = \tan 15^\circ$, and since α is acute, $\alpha = 15^\circ$