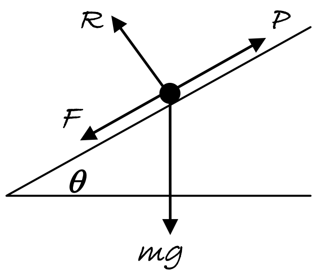
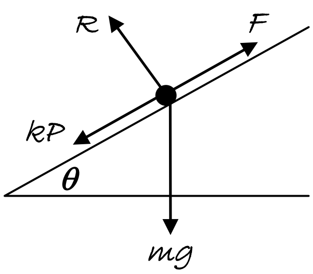
Section 1: Friction

Solutions to Exercise level 3

1.





Resolving perpendicular to plane (both cases) $R - mg\cos\theta = 0$

$R = mg\cos\theta$

Friction is limiting in both cases: $F = \mu R = \mu mg\cos\theta$

When the particle is about to move upwards (left-hand diagram)

Resolving up the plane: $P - F - mg\sin\theta = 0$

$P - \mu mg\cos\theta - mg\sin\theta = 0$

$P = \mu mg\cos\theta + mg\sin\theta$

When the particle is about to move downward (right-hand diagram)

Resolving down the plane: $kP + mg\sin\theta - F = 0$

$kP + mg\sin\theta - \mu mg\cos\theta = 0$

$kP = \mu mg\cos\theta - mg\sin\theta$

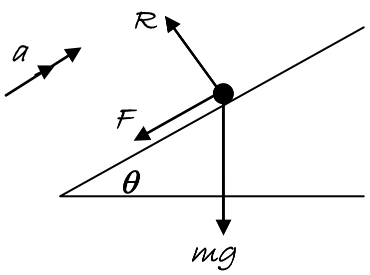
So $k(\mu mg\cos\theta + mg\sin\theta) = \mu mg\cos\theta - mg\sin\theta$

$k\mu\cos\theta + k\sin\theta = \mu\cos\theta - \sin\theta$

$(1+k)\sin\theta = \mu(1-k)\cos\theta$

$$\mu = \frac{1+k}{1-k}\tan\theta$$

2. (i) When moving upwards:



Resolving perpendicular to plane $R - mg\cos\theta = 0$

$R = mg\cos\theta$

Friction is limiting: $F = \mu R = \mu mg\cos\theta$

Resolving up the plane: $-F - mg\sin\theta = ma$

$$-\mu mg\cos\theta - mg\sin\theta = ma$$

$$a = -g(\mu\cos\theta + \sin\theta)$$

$$v = u + at$$

$$0 = u - g(\mu\cos\theta + \sin\theta)t$$

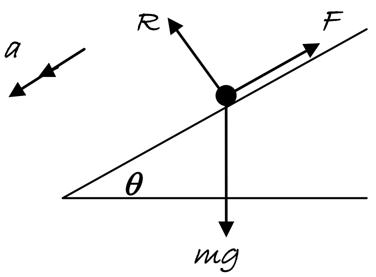
$$t = \frac{u}{g(\mu\cos\theta + \sin\theta)}$$

$$v^2 = u^2 + 2as$$

$$0 = u^2 - 2g(\mu\cos\theta + \sin\theta)s$$

$$s = \frac{u^2}{2g(\mu\cos\theta + \sin\theta)}$$

(ii) When moving down the plane:



Resolving down the plane: $mg\sin\theta - F = ma$

$$mg\sin\theta - \mu mg\cos\theta = ma$$

$$g\sin\theta - \mu g\cos\theta = a$$

For movement, $g\sin\theta - \mu g\cos\theta > 0$

$$\sin\theta > \mu\cos\theta$$

$$\tan\theta > \mu$$

(iii) $g(\sin\theta - \mu\cos\theta) = a$

$$s = ut + \tfrac{1}{2}at^2$$

$$\frac{u^2}{2g(\mu\cos\theta + \sin\theta)} = 0 + \tfrac{1}{2}g(\sin\theta - \mu\cos\theta)t^2$$

$$t^2 = \frac{u^2}{g^2(\sin\theta + \mu\cos\theta)(\sin\theta - \mu\cos\theta)}$$

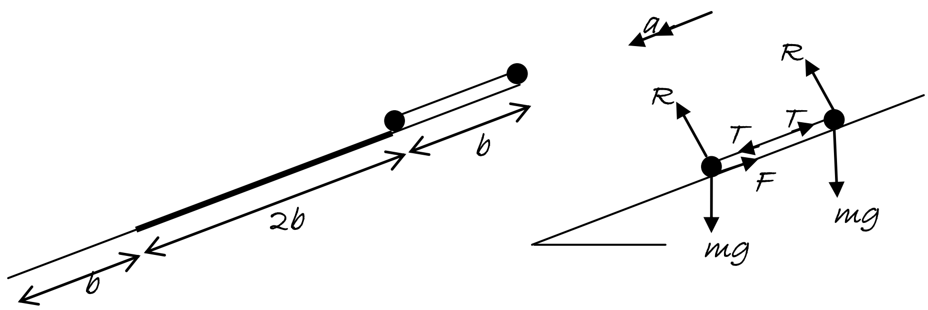
$$t^2 = \frac{u^2}{g^2(\sin^2\theta - \mu^2\cos^2\theta)}$$

$$t = \frac{u}{g\sqrt{\sin^2\theta - \mu^2\cos^2\theta}}$$

$$v = u + at$$
$$= 0 + g(\sin\theta - \mu\cos\theta) \times \frac{u}{g\sqrt{\sin^2\theta - \mu^2\cos^2\theta}}$$
$$= \frac{u(\sin\theta - \mu\cos\theta)}{\sqrt{\sin^2\theta - \mu^2\cos^2\theta}}$$
$$= \frac{u(\sin\theta - \mu\cos\theta)}{\sqrt{(\sin\theta - \mu\cos\theta)(\sin\theta + \mu\cos\theta)}}$$
$$= u\sqrt{\frac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta}}$$

3.

T



(i) For both particles: $R - mg\cos\theta = 0$

For lower particle: $F_{max} = \mu R = \mu mg\cos\theta$

For whole system: force down the slope $= 2mg\sin\theta$

For the system to move, this must be greater than the maximum friction force: $2mg\sin\theta > \mu mg\cos\theta$

$$\mu < 2\tan\theta$$

(ii) Stage 1: top particle on smooth and bottom particle on rough

Friction applies to just one particle:

$$2mg\sin\theta - \mu mg\cos\theta = ma_1$$
$$a_1 = g(2\sin\theta - \cos\theta)$$
$$u_1 = 0, s_1 = b$$
$$v^2 = u^2 + 2as$$
$$v_1^2 = 0 + 2g(2\sin\theta - \cos\theta)b$$
$$= 2gb(2\sin\theta - \cos\theta)$$

Stage 2: both particles on rough

Friction applies to both particles:

$$2mg\sin\theta - 2\mu mg\cos\theta = ma_2$$
$$a_2 = 2g(\sin\theta - \cos\theta)$$
$$u_2^2 = v_1^2 = 2gb(2\sin\theta - \cos\theta)$$
$$s_2 = b$$
$$v^2 = u^2 + 2as$$
$$\begin{aligned} v_2^2 &= 2gb(2\sin\theta - \cos\theta) + 2\times 2g(\sin\theta - \cos\theta)b \\ &= 2gb(2\sin\theta - \cos\theta + 2\sin\theta - 2\cos\theta) \\ &= 2gb(4\sin\theta - 3\cos\theta) \end{aligned}$$

Stage 3: top particle on rough and bottom particle on smooth
Friction applies to just one particle:
Same as stage 1 so $a_3 = a_1 = g(2\sin\theta - \cos\theta)$

$$u_3^2 = v_2^2 = 2gb(4\sin\theta - 3\cos\theta)$$
$$s_3 = b$$
$$v^2 = u^2 + 2as$$
$$\begin{aligned} v_3^2 &= 2gb(4\sin\theta - 3\cos\theta) + 2\times g(2\sin\theta - \cos\theta)b \\ &= 2gb(4\sin\theta - 3\cos\theta + 2\sin\theta - \cos\theta) \\ &= 2gb(6\sin\theta - 4\cos\theta) \\ &= 2gb(6\sin\theta - 4\cos\theta) \end{aligned}$$

so final speed is $= \sqrt{2gb(6\sin\theta - 4\cos\theta)}$