Section 1: The Normal distribution

Solutions to Exercise level 2

1. Distribution of scores is $X \sim N(100, 15^2)$

    (i) Probability of scoring less than 120 is $P(X < 119.5)$
    $P(X < 119.5) = 0.9032$

    (ii) Probability of scoring at least 110 is $P(X \geq 109.5)$
    $P(X \geq 109.5) = 0.2633$

    (iii) Probability of scoring between 85 and 105 inclusive is
    $P(84.5 \leq X < 105.5)$
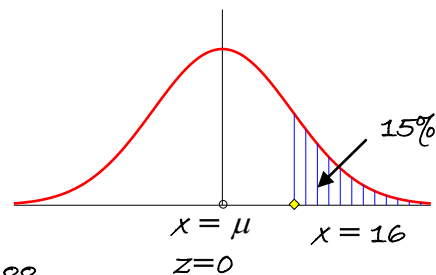 $P(84.5 \leq X < 105.5) = 0.4924$

2. $X \sim N(\mu, 3^2)$

    $Z = \dfrac{X - \mu}{3} \sim N(0,1)$

    $P(X > 16) = 0.15$

    $P(X < 16) = 0.85$

    $\Phi\left(\dfrac{16 - \mu}{3}\right) = 0.85 \quad \Rightarrow \dfrac{16 - \mu}{3} = 1.04 \quad \Rightarrow \mu = 12.88$

    $P(X > 12) = 0.6141$

3. $Z = \dfrac{X - \mu}{\sigma} \sim N(0,1)$

    $P(X > 428) = 0.18$

    $P(X < 428) = 0.82$

    $\Phi\left(\dfrac{428 - \mu}{\sigma}\right) = 0.82 \quad \Rightarrow \dfrac{428 - \mu}{\sigma} = 0.92$

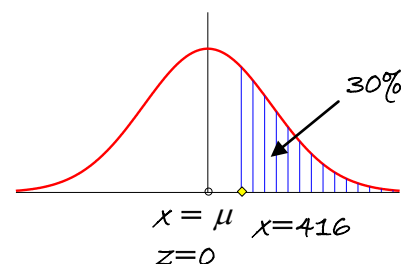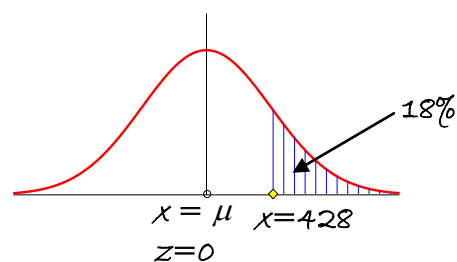 $P(X > 416) = 0.3$

    $P(X < 416) = 0.7$

    $\Phi\left(\dfrac{416 - \mu}{\sigma}\right) = 0.7 \quad \Rightarrow \dfrac{416 - \mu}{\sigma} = 0.52$

    $428 - \mu = 0.92\sigma$

    $416 - \mu = 0.52\sigma$

    Subtracting: $\quad 12 = 0.4\sigma$

    $\sigma = 30$

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$428 - \mu = 0.92 \times 30 \quad \Rightarrow \mu = 400.4$

The mean is 400 g and the standard deviation is 30 g.

4. $Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$

$P(X > 1230) = 0.07$

$P(X < 1230) = 0.93$
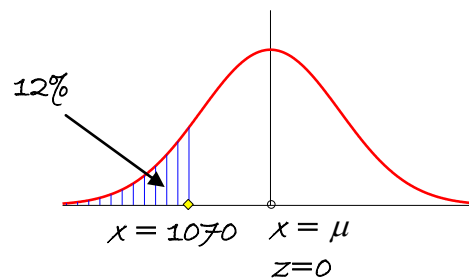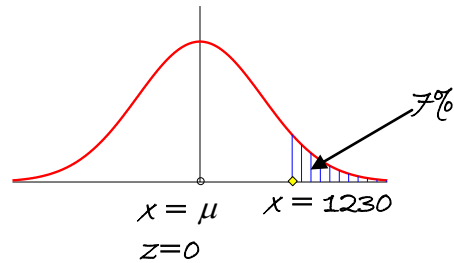
$\Phi\left(\dfrac{1230 - \mu}{\sigma}\right) = 0.93$

$\dfrac{1230 - \mu}{\sigma} = 1.48$

$P(X < 1070) = 0.12$

$\Phi\left(\dfrac{1070 - \mu}{\sigma}\right) = 0.12$

$\Phi\left(\dfrac{\mu - 1070}{\sigma}\right) = 0.88$

$\dfrac{\mu - 1070}{\sigma} = 1.175$

$1230 - \mu = 1.48\sigma$

$\mu - 1070 = 1.175\sigma$

Adding: $\quad 160 = 2.655\sigma$

$\sigma = 60.3$

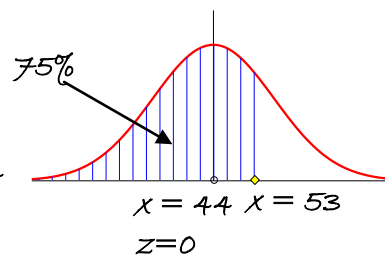$1230 - \mu = 1.48 \times 60.3 \quad \Rightarrow \mu = 1141$

The mean is 1141 hours and the standard deviation is 60.3 hours.

5. $Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$

By symmetry mean $= \dfrac{35 + 53}{2} = 44$

$P(X < 53) = 0.75$

$\Phi\left(\dfrac{53 - 44}{\sigma}\right) = 0.75 \quad \Rightarrow \dfrac{9}{\sigma} = 0.67 \quad \Rightarrow \sigma = 13.4$

The mean is 44 and the variance is 180.4.

6. $X \sim N(508.3, 4.8^2)$
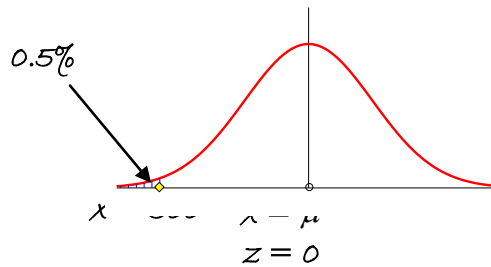
$P(X < 500) = 0.0418$

$P(X < 500) = 0.005$

$$\Phi\left(\frac{500-\mu}{4.8}\right) = 0.005$$

$$\Phi\left(\frac{\mu-500}{4.8}\right) = 0.995$$

$$\frac{\mu-500}{4.8} = 2.6$$

$$\mu = 512.5$$

0.5%

z = 0

7. (i) $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

$P(X > 30.6) = 0.3$

$P(X < 30.6 = 0.7$

$\Phi\left(\dfrac{30.6-\mu}{\sigma}\right) = 0.7$

$\dfrac{30.6-\mu}{\sigma} = 0.52$

$P(X < 29.2) = 0.15$

$\Phi\left(\dfrac{29.2-\mu}{\sigma}\right) = 0.15$

$\Phi\left(\dfrac{\mu-29.2}{\sigma}\right) = 0.85$

$\dfrac{\mu-29.2}{\sigma} = 1.04$

$$30.6 - \mu = 0.52\sigma$$

$$\mu - 29.2 = 1.04\sigma$$

Adding: $1.4 = 1.56\sigma$

$\sigma = 0.8974...$

$30.6 - \mu = 0.52 \times 0.8974... \Rightarrow \mu = 30.13...$

The mean is 30.1 cm and the standard deviation is 0.897 cm.

(ii) Using $\mu = 30.1$ and $\sigma = 0.897$

$P(X > 29.9) = 0.588$

(if using $\mu = 30.13$ and $\sigma = 0.8974$, $P(X > 29.9) = 0.601$)

(iii) $P(29.8 X < 30.5) = 0.303$

Percentage rejected = 69.7%