Section 1: Using the normal distribution

Solutions to Exercise level 2

1. $H_0: \mu = 500$
   $H_1: \mu < 500$
   where μ is the true population mean weight.
   $\bar{x} = \frac{7458}{15} = 497.2$

   One-tailed test at 2% significance level

   EITHER:

   from calculator using inverse normal of 0.02 for $N\left(500, \frac{5^2}{15}\right)$ gives critical value of 497.35, so critical region is $\bar{X} < 497.35$
   Sample mean is in the critical region, so reject H_0.
   The evidence suggests that the packets are underweight.

   OR:

   For $N\left(500, \frac{5^2}{15}\right)$, p-value $= P(\bar{X} < 497.2) = 0.015$
   p-value < 0.02, so reject H_0.
   The evidence suggests that the packets are underweight.

2. $H_0: \mu = 10.6$
   $H_1: \mu \neq 10.6$
   where μ is the true population mean length.

   Two-tailed test at 10% significance level

   EITHER:

   from calculator using inverse normal of 0.95 for $N\left(10.6, \frac{0.8}{50}\right)$ gives critical value of 10.81, so right-hand part of critical region is $\bar{X} > 10.81$
   Sample mean is not in the critical region, so accept H_0.
   There is not sufficient evidence to suggest that the mean length has changed, so the machine should not be recalibrated.

   OR:

   For $N\left(10.6, \frac{0.8}{50}\right)$, p-value $= P(\bar{X} > 10.79) = 0.067$
   p-value > 0.05, so accept H_0.

There is not sufficient evidence to suggest that the mean length has changed, so the machine should not be recalibrated.

3. (i) H_0: $\mu = 65$
H_1: $\mu < 65$
where μ is the population mean length of the metal rods.

For a one-tailed test at the 5% significance level
from calculator using inverse normal of 0.05 for $N\left(65, \frac{6^2}{25}\right)$ gives critical value of 63.03, so critical region is $\bar{X} < 63.03$

(ii) H_0: $\mu = 65$
H_1: $\mu \neq 65$
where μ is the population mean length of the metal rods.

For a two-tailed test at the 2% significance level
from calculator using inverse normal of 0.01 for $N\left(65, \frac{6^2}{25}\right)$ gives critical value of 62.21
and using inverse normal of 0.99 gives critical value of 67.79
so critical region is $\bar{X} < 62.21$ and $\bar{X} > 67.79$

4. (i) H_0: $\mu = 1050$
H_1: $\mu > 1050$
where μ is the population mean lifetime of the bulbs.

(ii) For $N\left(1050, \frac{69^2}{50}\right)$, p-value = $P(\bar{X} > 1065) = 0.062$

(iii) 1 tailed test at 5% significance level
p-value > 0.05, so accept H_0.
There is not sufficient evidence to suggest that the mean lifetime is greater than 1050 hours, so the manufacturer's claim is not justified at the 5% level.

5. H_0: $\mu = 11.9$
H_1: $\mu \neq 11.9$
where μ is the population mean breaking strength of the thread.

EITHER:

from calculator using inverse normal of 0.025 for $N\left(11.9, \frac{4.3}{40}\right)$ gives critical value of 11.26, so left-hand part of critical region is $\bar{X} < 11.26$
Sample mean is in the critical region, so reject H_0.
There is evidence to suggest that the mean breaking strength has changed.

OR:

For $N\left(11.9, \frac{4.3}{40}\right)$, p-value = $P(\bar{X} < 11.2) = 0.016$
p-value < 0.025, so reject H_0.
There is evidence to suggest that the mean breaking strength has changed.