## Section 1: Using the normal distribution

### Solutions to Exercise level 1

1. (i)   $H_0$: $\mu = 240$
   $H_1$: $\mu \neq 240$

   where $\mu$ is the population mean.

   $$\bar{X} \sim N\left(240, \frac{14^2}{20}\right)$$

   p-value $= P(\bar{X} > 247) = 0.0127$
   Two-tailed test with significance level 2%

   $2 \times 0.0127 > 0.02$, so accept $H_0$.
   There is not sufficient evidence to suggest that the population mean is different from 240.

   (ii)  $H_0$: $\mu = 165$
   $H_1$: $\mu > 165$

   where $\mu$ is the population mean.

   $$\bar{X} \sim N\left(165, \frac{15^2}{10}\right)$$

   p-value $= P(\bar{X} > 172.9) = 0.0479$
   One-tailed test at the 5% significance level,
   $0.0479 < 0.05$, so reject $H_0$.
   The evidence suggests that the population mean is greater than 165.

   (iii) $H_0$: $\mu = 240$
   $H_1$: $\mu < 240$

   where $\mu$ is the population mean.

   $$\bar{X} \sim N\left(240, \frac{24^2}{15}\right)$$

   p-value $= P(\bar{X} < 226) = 0.0119$
   One-tailed test at the 1% significance level
   $0.0119 > 0.01$, so accept $H_0$.
   There is not sufficient evidence to suggest that the population mean is less than 240.

2. $H_0$: $\mu = 67.4$
   $H_1$: $\mu < 67.4$

   where $\mu$ is the population mean weight of adult students.

   $$\bar{X} \sim N\left(67.4, \frac{3.8^2}{24}\right)$$

Using a calculator, inverse normal of 0.01 is 65.6

Critical region is $\bar{X} \leq 65.6$

The sample mean does not lie in the critical region, so accept $H_0$.

There is not sufficient evidence to suggest that the mean weight has decreased.

3. $H_0$: $\mu = 15$

   $H_1$: $\mu \neq 15$

   where $\mu$ is the population mean diameter of the ball bearings.

   $$\bar{X} \sim N\left( 15, \frac{1.41^2}{80} \right)$$

   Using a calculator, inverse normal of 0.01 is 14.63 and inverse normal of 0.99 is 15.37

   Critical region is $\bar{X} < 14.63$ and $\bar{X} > 15.37$

   The sample mean 14.64 does not lie in the critical region, so accept $H_0$.

   There is not sufficient evidence to suggest that the mean diameter has changed.