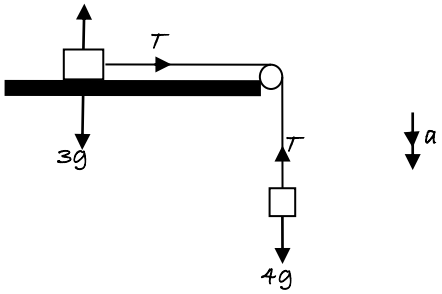
# Section 3: Connected objects

## Solutions to Exercise level 3

1. (i)




For particle X: $T = 3a$

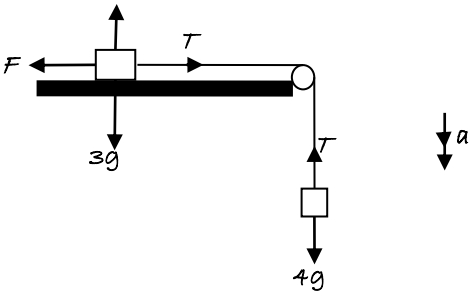
For particle Y: $4g - T = 4a$

$4g - 3a = 4a$

$7a = 4g$

$a = \frac{4}{7}g$

(ii)




For particle X: $T - F = 3 \times \frac{1}{2}g \quad \Rightarrow T = F + \frac{3}{2}g$

For particle Y: $4g - T = 4 \times \frac{1}{2}g$

$4g - F - \frac{3}{2}g = 2g$

$F = \frac{1}{2}g$

(iii) If the particle is about to move, $T = F = \frac{1}{2}g$, so mass for Y $= \frac{1}{2}$

So for motion, mass must be greater than $\frac{1}{2}$.

(iv) For particle X: $T = mb$

For particle Y: $ng - T = nb$

$ng - mb = nb$

$(m + n)b = ng$

$$b = \frac{n}{m+n}g$$

(v) Let mass of Y be $y$

$$\tfrac{1}{2}b = \frac{y}{m+y}g$$

$$\frac{n}{2(m+n)}g = \frac{y}{m+y}g$$

$$mn + ny = 2(m+n)y$$

$$mn = (2m+n)y$$

$$y = \frac{mn}{2m+n}$$

(vi) Let mass of X be $x$

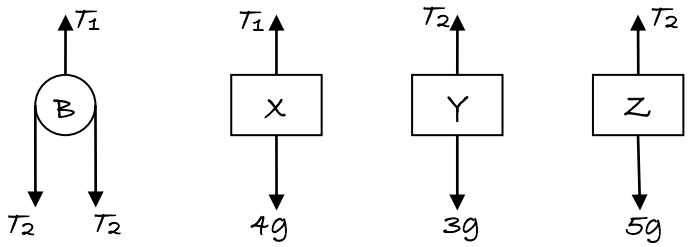
$$\tfrac{1}{2}b = \frac{n}{x+n}g$$

$$\frac{n}{2(m+n)}g = \frac{n}{x+n}g$$

$$2(m+n) = x+n$$

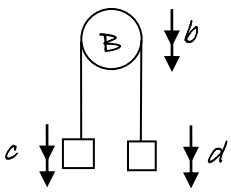
$$x = 2m+n$$

2. (i)




(ii) They are equal in magnitude but opposite in direction.

(iii)




Y is actually moving upwards, but we are measuring b, c and d downwards (so b and d will be positive numbers and c will be a negative number)

Acceleration of Y relative to B upwards is equal to acceleration of Z relative to B downwards.

$$c - b = -(d - b)$$

$$c - b = -d + b$$

$$c + d = 2b$$

(iv) For X: $T_1 - 4g = 4b \Rightarrow T_1 = 4g + 4b$ (1)

For Y: $3g - T_2 = 3c \Rightarrow T_2 = 3g - 3c$ (2)

For Z: $5g - T_2 = 5d \Rightarrow T_2 = 5g - 5d$ (3)

For B: $T_1 = 2T_2$ (since B has no mass) (4)

also from (iii) $c + d = 2b$ (5)

(2) and (3) give $3g - 3c = 5g - 5d$

$5d - 3c = 2g$

Using (5) gives $5(2b - c) - 3c = 2g$

$10b - 8c = 2g$

$5b - 4c = g$ (6)

Substituting (1) and (2) into (4) gives $4g + 4b = 2(3g - 3c)$

$2g + 2b = 3g - 3c$

$2b + 3c = g$ (7)

(6) and (7) give $\frac{5b - g}{4} = \frac{g - 2b}{3}$

$15b - 3g = 4g - 8b$

$23b = 7g$

$b = \frac{7}{23}g$

$c = \frac{1}{3}(g - \frac{14}{23}g) = \frac{3}{23}g$

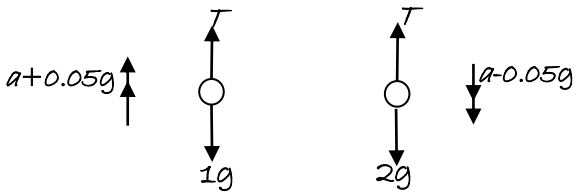
From (1) $T_1 = 4g + 4b = 4g + \frac{28}{23}g = \frac{120}{23}g$

So the acceleration of X is $\frac{7}{23}g$

the acceleration of Y is $\frac{3}{23}g$

and the tension in the string between A and B is $\frac{120}{23}g$

3. (i)




1 kg mass: $T - 1g = 1(a + 0.05g)$

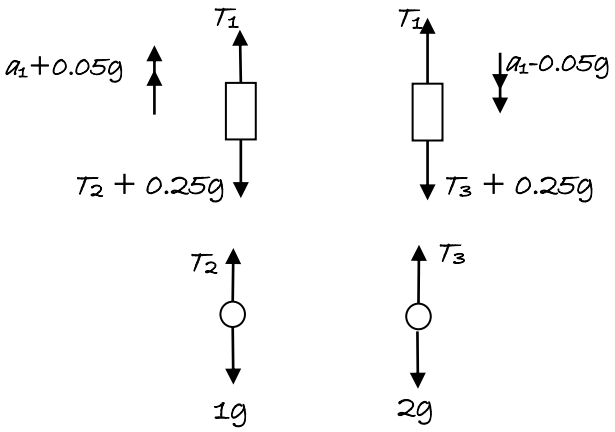
2 kg mass: $2g - T = 2(a - 0.05g)$

Adding: $g = 3a - 0.05g$

$3a = \frac{21}{20}g$

$a = \frac{7}{20}g$

(ii)




For 1 kg mass: $T_2 - 1g = 1(a_1 + \frac{1}{20}g)$

For left-hand spring: $T_1 - T_2 - \frac{1}{4}g = \frac{1}{4}(a_1 + \frac{1}{20}g)$

Adding: $T_1 - \frac{5}{4}g = \frac{5}{4}(a_1 + \frac{1}{20}g)$ (1)

For 2 kg mass: $2g - T_3 = 2(a_1 - \frac{1}{20}g)$

For right-hand spring: $T_3 + \frac{1}{4}g - T_1 = \frac{1}{4}(a_1 - \frac{1}{20}g)$

Adding: $\frac{9}{4}g - T_1 = \frac{9}{4}(a_1 - \frac{1}{20}g)$ (2)

(1) + (2) $g = \frac{14}{4}a_1 - \frac{1}{20}g$

$\frac{14}{4}a_1 = \frac{21}{20}g$

$a_1 = \frac{3}{10}g$

$T_2 = a_1 + \frac{21}{20}g = \frac{3}{10}g + \frac{21}{20}g = \frac{27}{20}g$

so the weight force registered by the 1 kg mass is $\frac{27}{20}g$

$T_3 = \frac{21}{10}g - 2a_1 = \frac{21}{10}g - \frac{6}{10}g = \frac{3}{2}g$

so the weight force registered by the 2 kg mass is $\frac{3}{2}g$