Section 3: The constant acceleration formulae

Solutions to Exercise level 3

1. (i) $s = 18, t = 1, a = -6, u = ?$

$s = ut + \frac{1}{2}at^2$

$18 = u + \frac{1}{2} \times -6 \times 1$

$18 = u - 3$

$u = 21$

(ii) $s = 18, t = ?, a = -6, u = 21$

$s = ut + \frac{1}{2}at^2$

$18 = 21t + \frac{1}{2} \times -6t^2$

$3t^2 - 21t + 18 = 0$

$t^2 - 7t + 6 = 0$

$(t-1)(t-6) = 0$

so the other time is after 6 seconds.

(iii) From (ii), $s = 21t - 3t^2$

$$= -3(t^2 - 7t)$$

$$= -3\left((t - \tfrac{7}{2})^2 - (\tfrac{7}{2})^2\right)$$

$$= -3(t - \tfrac{7}{2})^2 + \tfrac{147}{4}$$

Maximum positive displacement is $\frac{147}{4}$ or $36\frac{3}{4}$ metres (when $t = \frac{7}{2}$)

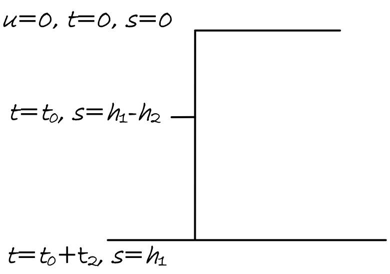
2. (i) Taking downwards as positive:

$s = h_1, u = 0, a = 10, t = t_1$

$s = ut + \frac{1}{2}at^2$

$h_1 = 5t_1^2$

(ii) Taking downwards as positive:



$$s = ut + \tfrac{1}{2}at^2 \Rightarrow \; h_1 - h_2 = 5t_0^2$$

$$h_1 = 5(t_0 + t_2)^2$$

$$\Rightarrow h_2 = 5(t_0 + t_2)^2 - 5t_0^2$$

$$\Rightarrow h_2 = 5t_0^2 + 10t_0t_2 + 5t_2^2 - 5t_0^2$$

$$\Rightarrow h_2 = 10t_0t_2 + 5t_2^2 = 5t_2(2t_0 + t_2)$$

$$\Rightarrow \frac{h_2}{5t_2} = 2t_0 + t_2$$

$$\Rightarrow t_0 = \frac{h_2}{10t_2} - \frac{t_2}{2}$$

$$h_1 = 5(t_0 + t_2)^2$$

$$= 5\left(\frac{h_2}{10t_2} - \frac{t_2}{2} + t_2\right)^2$$

$$= 5\left(\frac{h_2}{10t_2} + \frac{t_2}{2}\right)^2$$

3. (i) $100 \text{ mph} = \dfrac{100 \times 1600}{3600} = \dfrac{400}{9} \text{ ms}^{-1}$

$v = 0, a = ?, s = 1600, u = \dfrac{400}{9}$

$v^2 = u^2 + 2as$

$0 = \left(\dfrac{400}{9}\right)^2 + 2a \times 1600$

$a = -\dfrac{50}{81}$

The maximum deceleration is $\dfrac{50}{81} \approx 0.617 \text{ ms}^{-2}$.

(ii) $125 \text{ mph} = \dfrac{125 \times 1600}{3600} = \dfrac{500}{9} \text{ ms}^{-1}$

$v = 0, a_2 = ?, s = 1600, u = \dfrac{500}{9}$

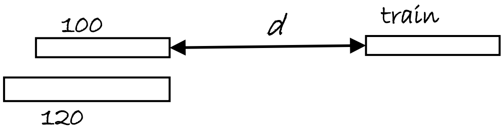
$v^2 = u^2 + 2as$

$0 = \left(\dfrac{500}{9}\right)^2 + 2a_2 \times 1600$

$a_2 = -\dfrac{625}{648}$

The deceleration is more than $\dfrac{625}{648} \approx 0.964 \text{ ms}^{-2}$.

4. (i)



$100 \text{ kmh}^{-1} = \frac{100000}{3600} = \frac{250}{9} \text{ ms}^{-1}$

$s = d + 100, u = \frac{250}{9}, v = 0, t = 30$

$s = \frac{1}{2}(u+v)t$

$d + 100 = \frac{1}{2}\left(\frac{250}{9} + 0\right) \times 30$

$d + 100 = \frac{1250}{3}$

$d = \frac{950}{3} = 316\frac{2}{3}$

The driver must start to brake 317 metres (3 s.f.) from the nearer end of the platform.

(ii) Maximum distance to travel $= d + 120 = 436\frac{2}{3} = \frac{1310}{3}$

$s = \frac{1310}{3}, u = \frac{250}{9}, v = 0, a = ?$

$v^2 = u^2 + 2as$

$0 = \left(\frac{250}{9}\right)^2 + 2a \times \frac{1310}{3}$

$a = -0.8835...$

so the deceleration must be at least 0.884 ms^{-2} (3 s.f.)

(iii) For the original acceleration: $u = \frac{250}{9}, v = 0, t = 30, a = ?$

$v = u + at$

$0 = \frac{250}{9} + 30t$

$t = -\frac{25}{27}$

For this new situation, $u = ?, v = 0, s = \frac{1310}{3}, a = -\frac{25}{27}$

$v^2 = u^2 + 2as$

$0 = u^2 + 2 \times \frac{1310}{3} \times -\frac{25}{27}$

$u = \sqrt{\frac{65500}{81}}$

$u = 28.4 \text{ ms}^{-1}$ (3 s.f.)

(iv) $u=\sqrt{\frac{65500}{81}}, v=0, s=\frac{1250}{3}, a=?$

$v^2=u^2+2as$

$0=\frac{65500}{81}+2a\times\frac{1250}{3}$

$a=-0.970 \text{ ms}^{-2}$ (3 s.f.)

Deceleration in this case is -0.970 ms^{-2} (3 s.f.)

5. (i) $u=u_0, v=0, a=-g, s=t$

$v=u+at$

$0=u_0-gt$

$t=\frac{u_0}{g}$

Time taken to maximum height $=\frac{u_0}{g}$ seconds.

(ii) Since the acceleration and distance are the same, it takes the same amount of time to come down as to go up, so time $=\frac{2u_0}{g}$ seconds.

(iii) Let initial speed of second stone be u_2.

Time to maximum height of second stone (measured from time that first stone reaches maximum height) $=\frac{u_2}{g}$

Let height of stones when they collide be h

For first stone, $s=h, a=-g, u=u_0, t=\frac{u_0}{g}+\frac{u_2}{g}$

$s=ut+\frac{1}{2}at^2$

$h=u_0\left(\frac{u_0}{g}+\frac{u_2}{g}\right)-\frac{g}{2}\left(\frac{u_0}{g}+\frac{u_2}{g}\right)^2$

For second stone, $s=h, a=-g, u=u_2, t=\frac{u_2}{g}$

$s=ut+\frac{1}{2}at^2$

$h=u_2\left(\frac{u_2}{g}\right)-\frac{g}{2}\left(\frac{u_2}{g}\right)^2=\frac{u_2^2}{g}-\frac{u_2^2}{2g}=\frac{u_2^2}{2g}$

$$\text{So } u_0\left(\frac{u_0}{g}+\frac{u_2}{g}\right)-\frac{g}{2}\left(\frac{u_0}{g}+\frac{u_2}{g}\right)^2=\frac{u_2^2}{2g}$$

$$2u_0(u_0+u_2)-(u_0+u_2)^2=u_2^2$$

$$2u_0^2+2u_0u_2-u_0^2-2u_0u_2-u_2^2=u_2^2$$

$$u_0^2=2u_2^2$$

$$u_2=\frac{1}{\sqrt{2}}u_0$$

(iv) The answer to (iii) would be unchanged as this does not depend on the value of g.