Section 3: The constant acceleration formulae

Solutions to Exercise level 2

1. $15\text{ kmh}^{-1} = \frac{15000}{3600}\text{ms}^{-1} = \frac{25}{6}\text{ms}^{-1}$

$30\text{ kmh}^{-1} = \frac{30000}{3600}\text{ms}^{-1} = \frac{25}{3}\text{ms}^{-1}$

$s = 1300$
$u = \frac{25}{6}$
$v = \frac{25}{3}$
$a = k$

$v^2 = u^2 + 2as$

$\left(\frac{25}{3}\right)^2 = \left(\frac{25}{6}\right)^2 + 2k \times 1300$

$\frac{625}{9} = \frac{625}{36} + 2600k$

$\frac{1875}{36} = 2600k$

$k = \frac{25}{1248}$

2. $108\text{ kmh}^{-1} = \frac{108000}{3600}\text{ms}^{-1} = 30\text{ms}^{-1}$

$u = 0$
$v = 30$
$s = 25$
$a = ?$

$v^2 = u^2 + 2as$

$30^2 = 0^2 + 2a \times 25$

$900 = 50a$

$a = 18$

The acceleration is 18 ms^{-2}.

Assumption: that the acceleration is constant.

3. While accelerating:

$u = 0$
$t = 20$
$v = 30$
$a = ?$

$v = u + at$

$30 = 0 + 20a$

$a = 1.5$

The acceleration in the first 20 seconds is 1.5 ms^{-2}

In the first 20 seconds

$u = 0$
$t = 20$
$v = 30$
$s = ?$

$s = \frac{1}{2}(u+v)t$

$= \frac{1}{2}(0+30) \times 20$

$= 300$

In the minute at constant speed of 30 ms^{-1}, distance $= 30 \times 60 = 1800$ m
Total distance travelled = 2100 m.

Assumption: that the acceleration is constant during the first 20 seconds.

4. $70 \text{ kmh}^{-1} = \frac{70000}{3600} ms^{-1} = \frac{175}{9} ms^{-1}$

$u = \frac{175}{9}$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = \left(\frac{175}{9}\right)^2 + 2 \times -5s$

$a = -5$

$s = ?$ $\quad s = 37.8$ (3 s.f.)

Yes, she comes to a stop 22.2 m before the accident.

5. $u = 20$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = 20^2 + 2a \times 30$

$s = 30$ $\quad 60a = -400$

$a = ?$ $\quad a = -6.67$

This is a deceleration of 6.67 ms^{-2}.

$u = 20$ $\quad s = \frac{1}{2}(u + v)t$

$v = 0$ $\quad 30 = \frac{1}{2}(0 + 20)t$

$s = 30$ $\quad 30 = 10t$

$t = ?$ $\quad t = 3$

It takes 3 seconds to come to rest.

6. $u = 2$ $\quad v = u + at$

$v = 16$ $\quad 16 = 2 + 10a$

$t = 10$ $\quad 14 = 10a$

$a = ?$ $\quad a = 1.4$

The acceleration is 1.4 ms^{-2}.

$u = 2$ $\quad s = \frac{1}{2}(u + v)t$

$v = 16$ $\quad = \frac{1}{2}(2 + 16) \times 10$

$t = 10$ $\quad = 90$

$s = ?$

It travels 90 m.

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7. Taking positive to be upwards:

$$\begin{aligned} u &= 6 \qquad & v &= u + at \\ a &= -9.8 & &= 6 - 9.8 \times 2 \\ t &= 2 & &= -13.6 \\ v &= ? \end{aligned}$$

The stone hits the water at a speed of 13.6 ms^{-1}.

$$\begin{aligned} u &= 6 \qquad & s &= ut + \tfrac{1}{2}at^2 \\ a &= -9.8 & &= 6 \times 2 + \tfrac{1}{2} \times -9.8 \times 2^2 \\ t &= 2 & &= -7.6 \\ s &= ? \end{aligned}$$

The initial height of the stone is 7.6 m.

8. Taking upwards to be positive:

$$\begin{aligned} u &= 25 \qquad & s &= ut + \tfrac{1}{2}at^2 \\ s &= 3 & 3 &= 25t + \tfrac{1}{2} \times -9.8t^2 \\ a &= -9.8 & 4.9t^2 - 25t + 3 &= 0 \\ t &= ? \end{aligned}$$

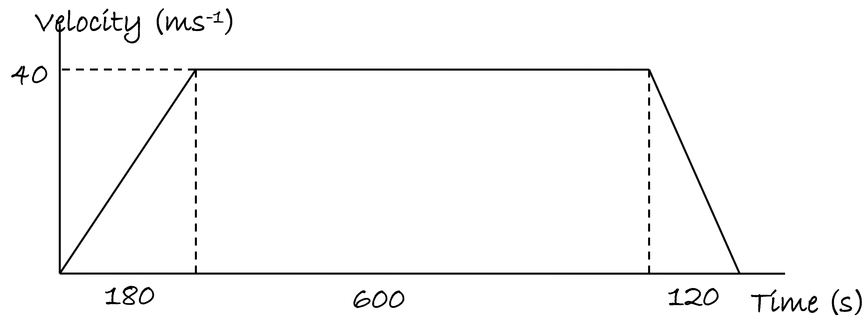
Using the quadratic formula:

$$t = \frac{25 \pm \sqrt{25^2 - 4 \times 4.9 \times 3}}{2 \times 4.9}$$

$$t = 0.123 \text{ or } 4.979$$

The ball is above 3 metres between these two times, so the length of time for which the ball is above 3 m is 4.979 – 0.123 = 4.856 seconds.

9. $144 \text{ kmh}^{-1} = \dfrac{144000}{60 \times 60} = 40 \text{ ms}^{-1}$



Total distance = area under graph

$$= \left(\tfrac{1}{2} \times 180 \times 40\right) + (600 \times 40) + \left(\tfrac{1}{2} \times 120 \times 40\right)$$

$$= 30000 \text{ m}$$

Distance between A and B = 30 km.

10. In the first two seconds:

$t = 2$ $\quad s = ut + \frac{1}{2}at^2$

$s = 10$ $\quad 10 = 2u + \frac{1}{2}a \times 2^2$

$u = ?$ $\quad 5 = u + a \qquad (1)$

$a = ?$

In the first four seconds:

$t = 4$ $\quad s = ut + \frac{1}{2}at^2$

$s = 32$ $\quad 32 = 4u + \frac{1}{2}a \times 4^2$

$u = ?$ $\quad 32 = 4u + 8a$

$a = ?$ $\quad 8 = u + 2a \qquad (2)$

(2) - (1) : $3 = a$

$u = 2$

In the first six seconds:

$t = 6$ $\quad s = ut + \frac{1}{2}at^2$

$u = 2$ $\quad = 2 \times 6 + \frac{1}{2} \times 3 \times 6^2$

$a = 3$ $\quad = 66$

$s = ?$

The distance moved in the last 2 seconds is 66 – 32 = 34 metres.

11. (i) For first 2 seconds:

$t = 2$ $\quad s = ut + \frac{1}{2}at^2$

$s = 30$ $\quad 30 = 2u + \frac{1}{2}a \times 2^2$

$u = ?$ $\quad 30 = 2u + 2a$

$a = ?$ $\quad 15 = u + a \qquad (1)$

For first 6 seconds:

$t = 6$ $\quad s = ut + \frac{1}{2}at^2$

$s = 60$ $\quad 60 = 6u + \frac{1}{2}a \times 6^2$

$u = ?$ $\quad 60 = 6u + 18a$

$a = ?$ $\quad 10 = u + 3a \qquad (2)$

Substituting (1) into (2): $10 = u + 3(15 - u)$

$10 = u + 45 - 3u$

$2u = 35$

$u = 17.5$

The initial velocity is 17.5 ms^{-1}.

(ii) From (1), $a = 15 - u = 15 - 17.5 = -2.5$

Deceleration = -2.5 ms^{-2}.

(iii) For the complete time to come to rest:

$u = 17.5 \qquad v = u + at$

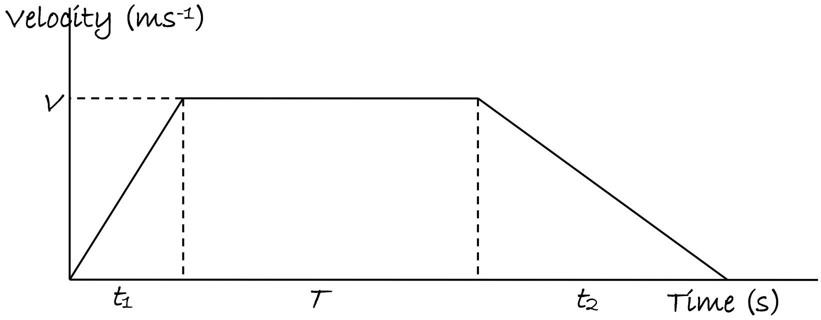
$a = -2.5 \qquad 0 = 17.5 - 2.5t$

$v = 0 \qquad t = 7$

$t = ?$

The total time is 7 seconds.

12.



During the acceleration: $3 = \frac{V}{t_1} \quad \Rightarrow t_1 = \frac{V}{3}$

During the deceleration: $1.5 = \frac{V}{t_2} \quad \Rightarrow t_2 = \frac{2V}{3}$

Total time = 60 seconds: $\frac{V}{3} + \frac{2V}{3} + T = 60$

$V + T = 60$

Distance travelled during acceleration $= \frac{1}{2}Vt_1 = \frac{1}{2}V \times \frac{1}{3}V = \frac{1}{6}V^2$

Distance travelled during constant speed $= VT$

Distance travelled during deceleration $= \frac{1}{2}Vt_2 = \frac{1}{2}V \times \frac{2}{3}V = \frac{1}{3}V^2$

Total distance = 1000 m: $\frac{1}{6}V^2 + VT + \frac{1}{3}V^2 = 1000$

$\frac{1}{2}V^2 + V(60 - V) = 1000$

$\frac{1}{2}V^2 + 60V - V^2 = 1000$

$\frac{1}{2}V^2 - 60V + 1000 = 0$

$V^2 - 120V + 2000 = 0$

$(V - 100)(V - 20) = 0$

$V = 100$ or $V = 20$

Since $V + T = 60$, V must be less than 60, so V = 20 ms^{-1}.

13. Taking upwards as positive:

For first ball:

$u = 25$ $\quad\quad s = ut + \frac{1}{2}at^2$

$a = -9.8$ $\quad\quad = 25 \times 2 + \frac{1}{2} \times -9.8 \times 2^2$

$t = 2$ $\quad\quad = 30.4$

$s = ?$

For second ball:

$u = -25$ $\quad\quad s = ut + \frac{1}{2}at^2$

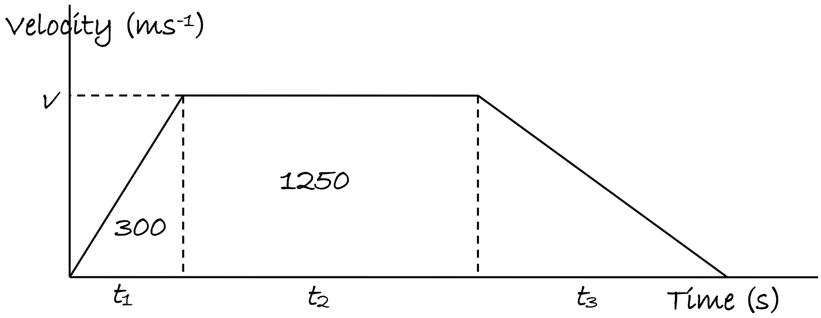
$a = -9.8$ $\quad\quad = -25 \times 2 + \frac{1}{2} \times -9.8 \times 2^2$

$t = 2$ $\quad\quad = -69.6$

$s = ?$

Distance between balls = 30.4 + 69.6 = 100 m.

14.



For acceleration: $\frac{1}{2}Vt_1 = 300 \quad \Rightarrow t_1 = \frac{600}{V}$

For constant speed: $Vt_2 = 1250 \quad \Rightarrow t_2 = \frac{1250}{V}$

$t_3 = \frac{2}{3}t_1 = \frac{400}{V}$

Total time = 3 minutes = 180 seconds

$$\frac{600}{V} + \frac{1250}{V} + \frac{400}{V} = 180$$

$$\frac{2250}{V} = 180$$

$$V = 12.5$$

$t_1 = \frac{600}{12.5} = 48$

$t_2 = \frac{1250}{12.5} = 100$

$t_3 = \frac{400}{12.5} = 32$

Distance during deceleration $= \frac{1}{2}Vt_3 = \frac{1}{2} \times 12.5 \times 32 = 200$

Total distance AB = 300 + 1250 + 200 = 1750 m.

15. (i) $u = 30$ $\quad s = ut + \frac{1}{2}at^2$

$a = -9.8$ $\quad 25 = 30t - 4.9t^2$

$s = 25$ $\quad 4.9t^2 - 30t + 25 = 0$

$t = ?$

$$t = \frac{30 \pm \sqrt{30^2 - 4 \times 4.9 \times 25}}{2 \times 4.9}$$

$t = 0.9950$ or 5.1274

Time it is above tower is $5.1274 - 0.9950 = 4.132$ s

(ii) Let h be the height above the top of the tower when they are at the same height:

For P_1: $u = 30$ $\quad s = ut + \frac{1}{2}at^2$

$a = -9.8$ $\quad 25 + h = 30t - 4.9t^2$

$s = 25 + h$ $\quad h = 30t - 4.9t^2 - 25$

$t = ?$

For P_2: $u = 10$ $\quad s = ut + \frac{1}{2}at^2$

$a = -9.8$ $\quad h = 10t - 4.9t^2$

$s = h$

$t = ?$

So $30t - 4.9t^2 - 25 = 10t - 4.9t^2$

$20t = 25$

$t = 1.25$

For P_1: $v = u + at$

$= 30 - 9.8 \times 1.25$

$= 17.75$

For P_2: $v = u + at$

$= 10 - 9.8 \times 1.25$

$= -2.25$

So the velocity of P_1 is 17.75 ms^{-1} and the velocity of P_2 is -2.25 ms^{-1}.

(iii) When P_1 is level with P_2, t = 1.25, and P_1 is moving upwards with speed 17.75 ms^{-1}, and P_2 is moving downwards.

At the point where P_1 stops moving upwards, measuring from the time when they are level:

$u = 17.75$ $\quad v = u + at$

$a = -9.8$ $\quad 0 = 17.75 - 9.8t$

$v = 0$ $\quad t = 1.81$

$t = ?$

So the time for which P_1 is higher than P_2 and is moving upwards is 1.81 s.