Section 2: Velocity and acceleration

Solutions to Exercise level 3

1. $100\text{km h}^{-1} = \dfrac{100000}{3600} = \dfrac{1000}{36}\text{m s}^{-1}$

   Average acceleration $= \dfrac{1000}{36 \times 8} = 3.47$ ms^{-2} (3 s.f.)

   $60\text{km h}^{-1} = \dfrac{60000}{3600} = \dfrac{600}{36}\text{m s}^{-1}$

   Average acceleration $= \dfrac{600}{36 \times 3} = 5.56$ ms^{-2} (3 s.f.)

   The comparison is not fair – in the second case the acceleration would not be sustainable.

   Increase in velocity $= 40\text{km h}^{-1} = \dfrac{40000}{3600} = \dfrac{400}{36}\text{m s}^{-1}$

   This would be over a time of 5 seconds

   Average acceleration $= \dfrac{400}{36 \times 5} = 2.22$ ms^{-2} (3 s.f.)

2. (i) $\dfrac{100\text{ kmh}^{-1}}{8\text{ seconds}} = 12.5\text{kmh}^{-1}\text{s}^{-1}$

   (ii) $12.5\text{kmh}^{-1}\text{s}^{-1} = \dfrac{12.5\text{kmh}^{-1}}{1\text{ sec}} = \dfrac{\frac{12.5\text{ km}}{1\text{ hour}}}{1\text{ sec}}$

   $= \dfrac{\frac{12.5\text{ km}}{1\text{ sec}}}{1\text{ hour}} = \dfrac{12.5\text{kms}^{-1}}{1\text{ hour}} = 12.5\text{kms}^{-1}\text{h}^{-1}$

   So it is the same.

3. (i) For acceleration, $V = m \times \frac{1}{6}T = \frac{1}{6}mT$

   Using area under graph, $D = 2 \times \frac{1}{2}V \times \frac{1}{6}T + V \times \frac{2}{3}T$

   $= \frac{5}{6}VT$

   $= \frac{5}{6} \times \frac{1}{6}mT \times T$

   $= \frac{5}{36}mT^2$

   (ii) Let the new time for accelerating and decelerating be kT.

   For acceleration, $V_1 = m \times kT = mkT$

Time at constant speed $= \frac{7}{9}T - 2kT$

Using area under graph, $D = 2 \times \frac{1}{2}V_1 \times kT + V_1(\frac{7}{9} - 2k)T$

$= (k + \frac{7}{9} - 2k)V_1T = (\frac{7}{9} - k) \times mkT^2$

This distance must be the same as in the previous case,

so $\frac{5}{36}mT^2 = (\frac{7}{9} - k)mkT^2$

$5 = 36(\frac{7}{9} - k)k$

$5 = 28k - 36k^2$

$36k^2 - 28k + 5 = 0$

$(2k - 1)(18k - 5) = 0$

$k = \frac{1}{2}$ or $\frac{5}{18}$

$k = \frac{1}{2}$ is not possible, since $2k < \frac{7}{9}$ so the time spent accelerating is $\frac{5}{18}T$

4. First competitor runs at speed 13 for t hours, and speed $(14 + \frac{2}{3}t)$ for (3 – t) hours.

Total distance $= 42.375$, so $42.375 = 13t + (14 + \frac{2}{3}t)(3 - t)$

$16t^2 - 24t + 9 = 0$

$(4t - 3)^2 = 0$

$t = \frac{3}{4}$

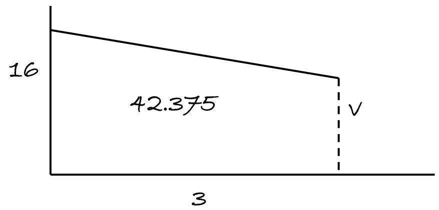
So speed for second stage is 14.5

For second competitor, constant deceleration starting from 16, time 3, distance 42.375

From area under graph,

$42.375 = \frac{1}{2}(16 + V) \times 3$

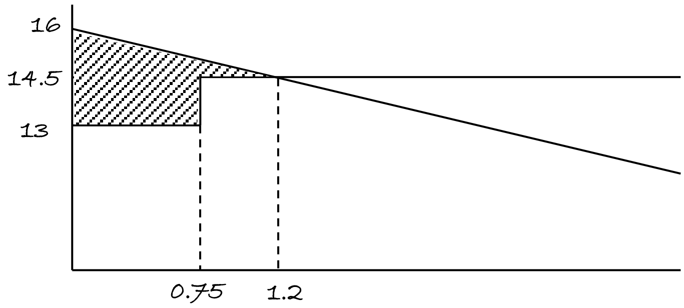
$V = 12.25$



Time at which speed = 14.5 (same as other competitor) is given by

$14.5 = 16 - 1.25t$

$t = 1.2$



Furthest distance apart is shaded area

Distance $= (\frac{1}{2} \times 1.5 \times 1.2) + (1.5 \times 0.75)$

$= 2.025$ km