Section 1: Displacement and distance

Solutions to Exercise level 3

1. The satellite takes 1 day to travel around the circumference of a circle with radius 42,164 km.

Circumference of circle $= 2\pi \times 42164$
Number of hours taken $= 24$

$$\text{Speed } = \frac{2\pi \times 42164}{24} = 11000 \text{ kmh}^{-1}$$

The velocity has magnitude 11000 kmh^{-1} and is directed along the tangent to the orbit.

Remember that velocity has direction as well as magnitude!

2. If the rocket accelerates at $3.5g$, the acceleration is 35 ms^{-1}.
The human body can survive this acceleration for 30 minutes = 1800 seconds.

$$\text{Acceleration } = \frac{\text{change in velocity}}{\text{time}} \Rightarrow \text{change in velocity} = \text{acceleration} \times \text{time}$$

so maximum change in velocity that is possible $= 35 \times 1800 = 63000$ ms^{-1}
so the human body can tolerate acceleration from rest to 63000 ms^{-1}.

$$\text{Escape velocity} = 40200 \text{ km h}^{-1} = \frac{40200 \times 1000}{3600} \text{ ms}^{-1}$$
$$= 11167 \text{ ms}^{-1}$$

This is less than the maximum change in velocity that the human body can tolerate, so it is well within human tolerance.

3. (i) Times taken in minutes are 25, 24, 25, 61, 29, 31, 55, 26
The fastest train is the 11 06.

$$\text{Average speed} = \frac{36}{24/60} = \frac{36 \times 60}{24} = 90 \text{ miles per hour}$$

(ii) The 10 42 is passed by the 11 00, the 11 06 and the 11 15.
The 10 57 is passed by the 11 00, the 11 06, the 11 15, the 11 18 and the 11 20.

For the 10 57: time taken = 55 mins so average speed $= \frac{36}{55}$ miles/min

For the 11 20: time taken = 31 mins so average speed $= \frac{36}{31}$ miles/min

11 20 leaves 23 minutes later
Let t be the time that they pass, measured in minutes from 11 20

Distance travelled by 10 57 is $(23+t) \times \frac{36}{55}$

Distance travelled by 11 20 is $t \times \frac{36}{31}$

$\frac{36}{55}(23+t)\times = \frac{36}{31}t$

$31(23+t) = 55t$

$713 = 24t$

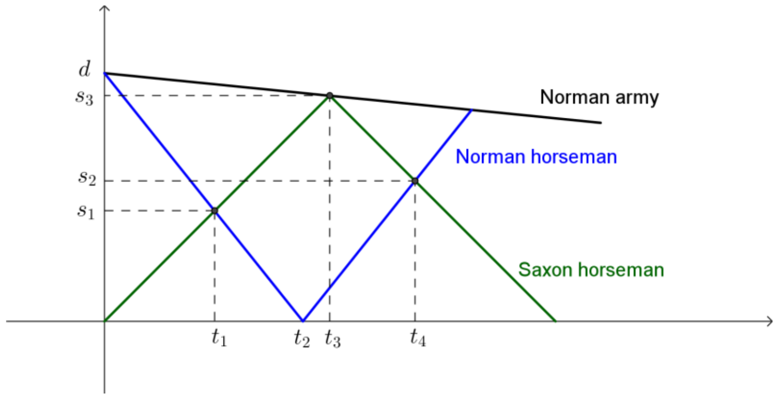
$29.7 = t$

Distance = 34.5

So they pass at about 11 50, about 1.5 miles from Reading.

The straight line model is not realistic as it does not take into account the times stopping at stations.

4.



At first pass Saxon horseman: $s_1 = xt_1$

Norman horseman: $d - s_1 = yt_1$

so $d - s_1 = y\frac{s_1}{x}$

$xd = xs_1 + ys_1$

$s_1 = \frac{xd}{x+y}$

So they pass each other at distance $\frac{xd}{x+y}$ from the Saxon army.

For Norman army: $d - s_3 = ut_3$ (1)

For Saxon reaching Norman army: $s_3 = xt_3$ (2)

(1) and (2) gives $\frac{d-s_3}{s_3} = \frac{u}{x} \Rightarrow xd - xs_3 = us_3 \Rightarrow s_3 = \frac{xd}{u+x}$ (3)

For Norman reaching Saxon army: $d = yt_2$ (4)

At second pass Saxon horseman: $s_3 - s_2 = x(t_4 - t_3)$ (5)

Norman horseman: $s_2 = y(t_4 - t_2)$ (6)

$s_3 - s_2 = x(t_4 - t_3) = xt_4 - xt_3$ (from (5))

$s_3 - s_2 = xt_4 - s_3$ (from (2))

$2s_3 - s_2 = xt_4$

$\dfrac{2xd}{u+x} - s_2 = xt_4$ (from (3))

$s_2 = yt_4 - yt_2 = yt_4 - d$ (from (6) and (4))

$s_2 + d = yt_4$

Hence $\dfrac{2xyd}{u+x} - s_2 y = s_2 x + dx$

$$s_2 x + s_2 y = \frac{2xyd}{u+x} - dx$$

$$s_2(x+y) = \frac{xd(2y-x-u)}{u+x}$$

$$s_2 = \frac{xd(2y-x-u)}{(x+y)(u+x)}$$

So they pass each other at distance $\dfrac{xd(2y-x-u)}{(x+y)(u+x)}$ from the Saxon army.

(i) $u > 2y - x \quad \Rightarrow x > 2y - u$

The speed of the Saxon is more than twice the speed of the Norman, so the Saxon will return to the Saxon line before the Norman reaches the Saxon line.

(ii) $u < y - 2x \quad \Rightarrow y > 2x - u$

The speed of the Norman is more than twice the speed of the Saxon, so the Norman will return to the Norman line before the Saxon reaches the Norman line.