## Section 1: Using calculus

### Solutions to Exercise level 3

1. (i) $s_1 - s_2 = t^3 + 5t^2 + 4t + 30 - (t^3 + 4t^2 + 7t + 5)$

$= t^2 - 3t + 25$

$= (t - \frac{3}{2})^2 - \frac{9}{4} + 25$

$= (t - \frac{3}{2})^2 + \frac{91}{4}$

This is at a minimum, so the particles are closest together, when $t = \frac{3}{2}$.

(ii) From (i), when $t = \frac{3}{2}$ their distance apart is $\frac{91}{4} = 22.75$

(iii) They are moving apart when $t > \frac{3}{2}$ (since $s_1 - s_2$ is increasing)

(iv) No, $s_1 - s_2$ continues to increase without limit.

2. (i) (a) The train accelerates uniformly for $0 < t < t_1$, then travels at a constant speed $v_1$ for $t_1 < t < t_2$, then decelerates uniformly for $t_2 < t < t_3$.

When $t = t_1$ $v = v_1$
When $t = t_2$ $v = v_1$
When $t = t_3$ $v = 0$

(b) For $0 < t < t_1$ $\quad v = at = \dfrac{v_1 t}{t_1}$

For $t_1 < t < t_2$ $\quad v = v_1$

For $t_2 < t < t_3$ $\quad v = v_1 - at$

$$= v_1 - \frac{v_1}{t_3 - t_2}(t - t_2)$$

$$= v_1\left(\frac{t_3 - t_2 - t + t_2}{t_3 - t_2}\right)$$

$$= v_1\left(\frac{t_3 - t}{t_3 - t_2}\right)$$

(c) For $0 < t < t_1$ $\quad$ displacement $= \frac{1}{2}v_1 t_1$

For $t_1 < t < t_2$ $\quad$ displacement $= v_1(t_2 - t_1)$

For $t_2 < t < t_3$ $\quad$ displacement $= \frac{1}{2}v_1(t_3 - t_2)$

(ii) For $0 < t < t_1$

$$\frac{dv}{dt} = At(t - t_1) = A(t^2 - t_1 t)$$

$$v = A(\tfrac{1}{3}t^3 - \tfrac{1}{2}t_1 t^2) + c_1$$

When $t = 0, v = 0 \Rightarrow c_1 = 0$

When $t = t_1, v = V_1 \Rightarrow V_1 = A(\tfrac{1}{3}t_1^3 - \tfrac{1}{2}t_1^3) = -\tfrac{1}{6}At_1^3$

$$\Rightarrow A = -\frac{6V_1}{t_1^3}$$

$$v = -\frac{6V_1}{t_1^3}\left(\tfrac{1}{3}t^3 - \tfrac{1}{2}t_1 t^2\right)$$

$$= \frac{V_1 t^2}{t_1^3}\left(3t_1 - 2t\right)$$

For $t_1 < t < t_2$

$$\frac{dv}{dt} = 0 \Rightarrow v = c_2$$

When $t = t_1, v = V_1 \Rightarrow c_2 = V_1$

so $v = V_1$

For $t_2 < t < t_3$

$$\frac{dv}{dt} = B(t - t_2)(t - t_3) = B(t^2 - (t_2 + t_3)t + t_2 t_3)$$

$$v = B(\tfrac{1}{3}t^3 - \tfrac{1}{2}(t_2 + t_3)t^2 + t_2 t_3 t) + c_3$$

When $t = t_3, v = 0$

$$0 = B(\tfrac{1}{3}t_3^3 - \tfrac{1}{2}(t_2 + t_3)t_3^2 + t_2 t_3^2) + c_3$$

$$0 = B(-\tfrac{1}{6}t_3^3 + \tfrac{1}{2}t_2 t_3^2) + c_3$$

$$0 = \tfrac{1}{6}Bt_3^2(3t_2 - t_3) + c_3 \qquad (1)$$

When $t = t_2, v = V_1$

$$V_1 = B(\tfrac{1}{3}t_2^3 - \tfrac{1}{2}t_2^3 - \tfrac{1}{2}t_3 t_2^2 + t_2^2 t_3) + c_3$$

$$= B(-\tfrac{1}{6}t_2^3 + \tfrac{1}{2}t_2^2 t_3) + c_3$$

$$= \tfrac{1}{6}Bt_2^2(3t_3 - t_2) + c_3 \qquad (2)$$

(2) – (1):

$$V_1 = \tfrac{1}{6}Bt_2^2(3t_3 - t_2) - \tfrac{1}{6}Bt_3^2(3t_2 - t_3)$$

$$= \tfrac{1}{6}B(3t_2^2 t_3 - t_2^3 - 3t_2 t_3^2 + t_3^3)$$

$$= \tfrac{1}{6}B(t_3 - t_2)^3$$

$$B = \frac{6V_1}{(t_3 - t_2)^3}$$

Substituting into (1):

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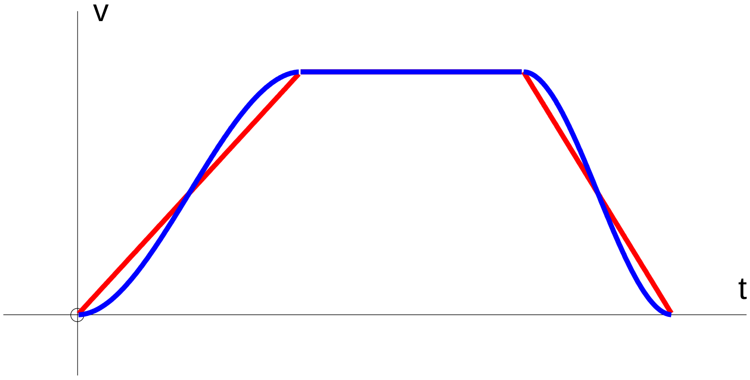
$$0 = \tfrac{1}{6} \times \frac{6V_1}{(t_3 - t_2)^3} t_3^2 (t_3 - 3t_2) + c_3$$

$$c_3 = \frac{V_1 t_3^2 (t_3 - 3t_2)}{(t_3 - t_2)^3}$$

$$\text{so } \; v = \frac{6V_1}{(t_3 - t_2)^3}\left(\tfrac{1}{3}t^3 - \tfrac{1}{2}(t_2 + t_3)t^2 + t_2 t_3 t\right) + \frac{V_1 t_3^2 (t_3 - 3t_2)}{(t_3 - t_2)^3}$$

$$= \frac{V_1}{(t_3 - t_2)^3}\left(2t^3 - 3(t_2 + t_3)t^2 + 6t_2 t_3 t + t_3^2 (t_3 - 3t_2)\right)$$

$$= \frac{V_1}{(t_3 - t_2)^3}\left(2t^3 - 3(t_2 + t_3)t^2 + 6t_2 t_3 t + t_3^2 (t_3 - 3t_2)\right)$$




The new model may be more realistic – it shows smoother transitions.

3. (i) For the second model, $v = \dfrac{V_1 t^2}{t_1^3}\left(3t_1 - 2t\right)$

$$s = \int_0^{t_1} \left( \frac{V_1 t^2}{t_1^3}\left(3t_1 - 2t\right) \right) dt$$

$$= \frac{V_1}{t_1^3} \int_0^{t_1} \left(3t_1 t^2 - 2t^3\right) dt$$

$$= \frac{V_1}{t_1^3} \left[ t_1 t^3 - \tfrac{1}{2} t^4 \right]_0^{t_1}$$

$$= \frac{V_1}{t_1^3} (t_1^4 - \tfrac{1}{2} t_1^4)$$

$$= \frac{V_1 t_1}{2}$$

so displacement is the same.

(ii) $\dfrac{dv}{dt} = -\dfrac{6V_1}{t_1^3}(t^2 - t_1 t)$

$= -\dfrac{6V_1}{t_1^3}\left((t - \tfrac{1}{2}t_1)^2 - \tfrac{1}{4}t_1^2\right)$

So maximum acceleration is $\dfrac{6V_1}{t_1^3} \times \tfrac{1}{4}t_1^2 = \dfrac{3V_1}{2t_1}$

so it is $\tfrac{3}{2}\times$the acceleration on the original model.