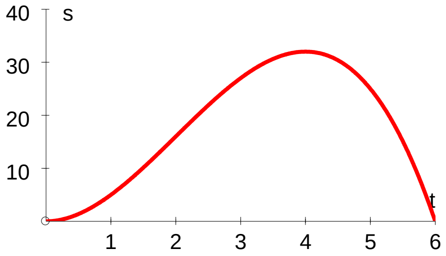
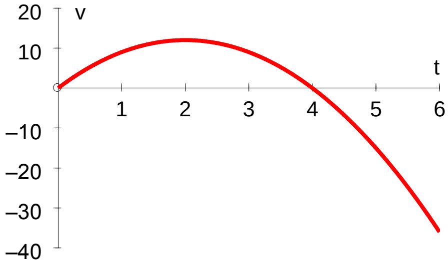
Section 1: Using calculus

Solutions to Exercise level 2

1. (i) $s = 6t^2 - t^3 = t^2(6-t)$



$v = \frac{ds}{dt} = 12t - 3t^2 = 3t(4-t)$



(ii) The particle is at O when $t = 0$ and when $t = 6$.

(iii) The greatest displacement is when the velocity is zero.

$3t(4-t) = 0$

$t = 0$ or $t = 4$

From the graph, the greatest displacement is when $t = 4$

$s = 4^2(6-4) = 32$

The greatest displacement is 32 m.

(iv) The greatest positive speed is when the acceleration is zero

$a = \frac{dv}{dt} = 12 - 6t$

When $a = 0$, $t = 2$

When $t = 2$, $v = 3 \times 2(4-2) = 12$

The greatest negative speed in the time interval is when $t = 6$ (from the graph).

When $t = 6$, $v = 3 \times 6(4-6) = -36$

So the greatest speed in the time interval is 36 ms^{-1}.

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2. (i) $s = t^3 - 12t$

$v = \frac{ds}{dt} = 3t^2 - 12$

When $t = 0$, $v = -12$, so the initial velocity is -12 ms^{-1}.

When $t = 0$, $s = 0$

When $t = 4$, $s = 4^3 - 12 \times 4 = 16$

The difference between its positions is 16 m.

(ii) When $t = 0$, $v = -12$

When $t = 4$, $v = 3 \times 4^2 - 12 = 36$

The velocity is initially negative, but when $t = 4$ the velocity is positive, so the particle has changed direction.

(iii) When $v = 0$, $3t^2 - 12 = 0$

$t^2 = 4$

$t = \pm 2$

Since t must be positive, the particle changes direction when $t = 2$

When $t = 2$, $s = 2^3 - 12 \times 2 = -16$

So between $t = 0$ and $t = 2$, the particle travels from $s = 0$ to $s = -16$, so it travels 16 m. Between $t = 2$ and $t = 4$, it travels from $s = -16$ to $s = 16$, so it travels 32 m. So the total distance travelled is 48 m.

3. $a = 6 - 2t$

$v = \int (6 - 2t)\,dt = 6t - t^2 + c$

When $t = 0$, $v = 0 \Rightarrow c = 0$

$v = 6t - t^2 = t(6 - t)$

The vehicle comes to rest again when $t = 6$, so it reaches point B when $t = 6$.

$s = \int \left(6t - t^2\right) dt = 3t^2 - \frac{1}{3}t^3 + k$

When $t = 0$, $s = 0 \Rightarrow k = 0$

$s = 3t^2 - \frac{1}{3}t^3$

When $t = 6$, $s = 3 \times 6^2 - \frac{1}{3} \times 6^3 = 36$

The distance AB is 36 m.

At greatest speed, acceleration is zero $\Rightarrow t = 3$

When $t = 3$, $v = 3(6 - 3) = 9$

The greatest speed is 9 ms^{-1}.

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4. (i) $a = 6t - 4$

$v = \int (6t-4)\,dt = 3t^2 - 4t + c$

When $t = 0$, $v = 0 \Rightarrow c = 0$

$v = 3t^2 - 4t$

$s = \int \left(3t^2 - 4t\right) dt = t^3 - 2t^2 + k$

When $t = 0$, $s = 0 \Rightarrow k = 0$

$s = t^3 - 2t^2$

(ii) When $s = 0$, $t^3 - 2t^2 = 0$

$t^2(t-2) = 0$

$t = 0$ or $t = 2$

The particle is at the origin when $t = 0$ and when $t = 2$.

(iii) The particle changes direction when $v = 0 \Rightarrow 3t^2 - 4t = 0$

$\Rightarrow t(3t-4) = 0$

$\Rightarrow t = 0$ or $t = \frac{4}{3}$

The particle does not change direction in the first second.

When $t = 1$, $s = 1^3 - 2\times 1^2 = -1$

so the distance travelled in the first second is 1 m.

5. (i) $v = 12t^2 - 4t^3 = 4t^2(3-t)$

When $v = 0$, $t = 0$ or $t = 3$.

$s = \int \left(12t^2 - 4t^3\right) dt = 4t^3 - t^4 + c$

When $t = 0$, $s = 0 \Rightarrow c = 0$

$s = 4t^3 - t^4$

When the particle is next at rest, $t = 3$ so $s = 4\times 3^3 - 3^4 = 27$

The distance travelled is 27 m.

(ii) $a = \dfrac{dv}{dt} = 24t - 12t^2 = 12t(2-t)$

By symmetry the greatest acceleration is when $t = 1$

The greatest acceleration is 12 ms^{-2}.

(iii) The greatest speed is when the acceleration is zero

$12t(2-t) = 0$

$t = 0$ or $t = 2$

When $t = 0$, $v = 0$

When $t = 2$, $v = 4\times 2^2(3-2) = 16$

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6. $a = k(1+3t^2)$

When $t = 3$, $a = 14 \Rightarrow 14 = k(1+3\times 3^2)$

$\Rightarrow 14 = 28k$

$\Rightarrow k = \frac{1}{2}$

$a = \frac{1}{2}(1+3t^2)$

$v = \int \frac{1}{2}(1+3t^2)dt = \frac{1}{2}t + \frac{1}{2}t^3 + c$

When $t = 3$, $v = 25 \Rightarrow 25 = \frac{1}{2}\times 3 + \frac{1}{2}\times 3^3 + c$

$\Rightarrow 25 = 15 + c$

$\Rightarrow c = 10$

$v = \frac{1}{2}t + \frac{1}{2}t^3 + 10$

Initial velocity = 10 ms^{-1}.

7. $s = 41 + t^3 + t^2 + 12t$

$v = \frac{ds}{dt} = 3t^2 + 2t + 12$

If there is a change of direction, then the velocity is zero,

$3t^2 + 2t + 12 = 0$

For this quadratic equation, the discriminant "$b^2 - 4ac$" is

$2^2 - 4\times 3\times 12 = -140$. Since the discriminant is negative, the equation has no real solutions and so the velocity is never zero. Therefore the particle never changes its direction of motion.

8. (i) $s = 2t^3 - 3t$

$v = \frac{ds}{dt} = 6t^2 - 3$

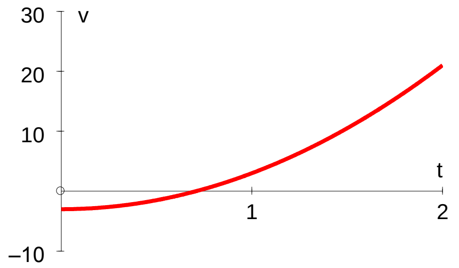
$a = \frac{dv}{dt} = 12t$

(ii) When $v = 0$, $6t^2 - 3 = 0$

$t^2 = \frac{1}{2}$

$t = \frac{1}{\sqrt{2}} = 0.707$ (since t must be positive)

(iii) $v = 6t^2 - 3$



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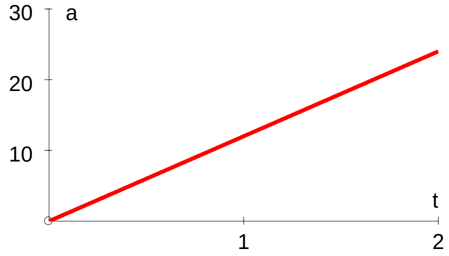
$a = 12t$



(iv) The object moves towards P until $t = 0.707$. At this point it comes to rest instantaneously and then starts to accelerate towards O. It passes O and when $t = 2$ it is accelerating towards Q.

(v) When $t = 2,\ s = 2\times 2^3 - 3\times 2 = 10$
The displacement when t = 2 is 10 m.

When $t = 0,\ s = 0$

When it comes to rest, $t = \frac{1}{\sqrt{2}}$, so $s = 2\times\left(\frac{1}{\sqrt{2}}\right)^3 - 3\times\frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

When t = 2, s = 10

Total distance travelled $= 2\sqrt{2} + 10 = 12.8$ m.

9. $a = 24t - 12t^2$

(i) $v = \int a\,dt = \int (24t - 12t^2)\,dt = 12t^2 - 4t^3 + c$

When $t = 0, v = 0 \Rightarrow c = 0$

$v = 12t^2 - 4t^3$

$s = \int v\,dt = \int (12t^2 - 4t^3)\,dt = 4t^3 - t^4 + d$

When $t = 0, s = 0 \Rightarrow d = 0$

$s = 4t^3 - t^4$

When it returns to O, $4t^3 - t^4 = 0$

$t^3(4 - t) = 0$

$t = 0$ or 4

So it returns to O when $t = 4$ s

When $t = 4,\ v = 12\times 16 - 4\times 64 = -64$
so its velocity at this time is -64 ms^{-1}.

(ii) Maximum displacement is when $v = 0$

$12t^2 - 4t^3 = 0$

$3t^2 - t^3 = 0$

$t^2(3 - t) = 0$

so maximum displacement is when $t = 3$

$s = 4\times 27 - 81 = 27$

Maximum displacement = 27 m

(iii) Maximum velocity is when $a = 0$

$24t - 12t^2 = 0$

$2t - t^2 = 0$

$t(t - 2) = 0$

so maximum velocity is when $t = 2$

$v = 12 \times 4 - 4 \times 8 = 16$

Its maximum velocity is 16 ms^{-1}.

Its greatest speed is 64 ms^{-1} (from part (ii), when $t = 4$)