Section 1: Using calculus

Solutions to Exercise level 1

1. (a) $s = 5 + 3t - 2t^2$

   (i) $v = \dfrac{ds}{dt} = 3 - 4t$

   (ii) When $t = 0$, $s = 5, v = 3$

   (iii) When $v = 0$, $3 - 4t = 0 \Rightarrow t = \frac{3}{4} = 0.75$

   When $t = \frac{3}{4}$, $s = 5 + 3 \times \frac{3}{4} - 2 \times \frac{9}{16} = 6.125$

   (b) $s = 4t^2 - 3t$

   (i) $v = \dfrac{ds}{dt} = 8t - 3$

   (ii) When $t = 0$, $s = 0, v = -3$

   (iii) When $v = 0$, $8t - 3 = 0 \Rightarrow t = \frac{3}{8} = 0.375$ s

   When $t = \frac{3}{8}$, $s = 4 \times \frac{9}{64} - 3 \times \frac{3}{8} = -0.5625$

   (c) $s = 2t^3 - 4t^2 - 8t + 3$

   (i) $v = \dfrac{ds}{dt} = 6t^2 - 8t - 8$

   (ii) When $t = 0$, $s = 3, v = -8$

   (iii) When $v = 0$, $6t^2 - 8t - 8 = 0$

   $\Rightarrow 3t^2 - 4t - 4 = 0$

   $\Rightarrow (3t + 2)(t - 2) = 0$

   Since t must be positive, $t = 2$

   When $t = 2$, $s = 2 \times 8 - 4 \times 4 - 8 \times 2 + 3 = -13$

2. (a) $v = 3t + 2$

   (i) $a = \dfrac{dv}{dt} = 3$

   (ii) When $t = 0$, $v = 2, a = 3$

   (b) $v = 4t^2 - 3t + 5$

   (i) $a = \dfrac{dv}{dt} = 8t - 3$

   (ii) When $t = 0$, $v = 5, a = -3$

   (c) $v = 4t^3 - 3t^2 + 5$

(i) $a = \frac{dv}{dt} = 12t^2 - 6t$

(ii) When $t = 0$, $v = 5, a = 0$

3. (i) $x = \int v\, dt = \int (3t+2)\, dt = \frac{3}{2}t^2 + 2t + c$

When $t = 0, x = 5 \Rightarrow 5 = c$

$x = \frac{3}{2}t^2 + 2t + 5$

(ii) When $v = 26$, $26 = 3t + 2 \Rightarrow 3t = 24 \Rightarrow t = 8$

When $t = 8$, $x = \frac{3}{2} \times 64 + 2 \times 8 + 5 = 117$

(iii) When $x = 10$, $10 = \frac{3}{2}t^2 + 2t + 5$

$$\frac{3}{2}t^2 + 2t - 5 = 0$$

$$3t^2 + 4t - 10 = 0$$

$$t = \frac{-4 \pm \sqrt{16 - 4 \times 3 \times -10}}{6}$$

Since t is positive, $t = \frac{-4 + \sqrt{136}}{6} = \frac{-2 + \sqrt{34}}{3}$

$$t = 1.277$$

For this value of t, $v = 3\left(\frac{-2 + \sqrt{34}}{3}\right) + 2 = \sqrt{34} = 5.83$ (3 s.f.)

(iv) $a = \frac{dv}{dt} = 3$

4. $s = t^3 + 2t^2 + 3t + 4$

$v = \frac{ds}{dt} = 3t^2 + 4t + 3$

$a = \frac{dv}{dt} = 6t + 4$

When $t = 2$, $v = 3 \times 2^2 + 4 \times 2 + 3 = 23$

$a = 6 \times 2 + 4 = 16$

5. (i) When $t = 0$, $s = -2$, so the initial displacement = -2 m.

$s = 2t^2 + 3t - 2$

$v = \frac{ds}{dt} = 4t + 3$

When $t = 0$, $v = 3$ so the initial velocity is 3 ms^{-1}.

(ii) $v = 0 \quad \Rightarrow 4t + 3 = 0 \quad \Rightarrow t = -0.75$

Since this is negative, there are no times for which the velocity is zero.

(iii) When $s = 0$, $2t^2 + 3t - 2 = 0$

$$(2t - 1)(t + 2) = 0$$

$$t = \tfrac{1}{2} \text{ or } -2$$

The particle is at the origin when $t = \tfrac{1}{2}$.

6. (i) $s = t^3 - 2t^2 - 15t$

$$v = \frac{ds}{dt} = 3t^2 - 4t - 15$$

When $t = 0$, $v = -15$ so the initial velocity is -15 ms^{-1}.

$$a = \frac{dv}{dt} = 6t - 4$$

When $t = 0$, $a = -4$ so the initial acceleration is -4 ms^{-2}.

(ii) When $v = 0$, $3t^2 - 4t - 15 = 0$

$$(3t + 5)(t - 3) = 0$$

$$t = -\tfrac{5}{3} \text{ or } 3$$

The velocity is zero after 3 seconds.

(iii) When the velocity is at its minimum value, $a = 0$

$$\Rightarrow 6t - 4 = 0$$

$$\Rightarrow t = \tfrac{2}{3}$$

When $t = \tfrac{2}{3}$, $v = 3\left(\tfrac{2}{3}\right)^2 - 4 \times \tfrac{2}{3} - 15 = -\tfrac{49}{3}$.

(This must be a minimum point since the equation of the velocity is a quadratic with positive coefficient of x^2.)

7. $v = 2t^3 - 9t^2$

$$s = \int \left(2t^3 - 9t^2\right) dt = \tfrac{1}{2}t^4 - 3t^3 + c$$

When $t = 0$, $s = 20 \Rightarrow 20 = c$

$$s = \tfrac{1}{2}t^4 - 3t^3 + 20$$

$$a = \frac{dv}{dt} = 6t^2 - 18t$$

When acceleration is zero, $6t^2 - 18t = 0$

$$6t(t-3) = 0$$

$$t = 0 \text{ or } t = 3$$

The acceleration is zero when $t = 0$ and when $t = 3$.

8. $v = 8t - 3t^2$

(i) When $t = 2$, $v = 8 \times 2 - 3 \times 4 = 4$ ms^{-1}.

(ii) $a = \dfrac{dv}{dy} = 8 - 6t$

(iii) When $t = 3$, $a = 8 - 6 \times 3 = -10$ ms^{-2}.

(iv) $s = \int v\,dt = \int (8t - 3t^2)\,dt = 4t^2 - t^3 + c$

Since initial position is 0, $c = 0$

$s = 4t^2 - t^3$

(v) When $t = 3$, $s = 4 \times 9 - 27 = 9$

so it is 9 m from O.

9. $a = 3 + 2t$

$v = \int (3 + 2t)\,dt = 3t + t^2 + c$

When $t = 2$, $v = 10 \Rightarrow 10 = 3 \times 2 + 2^2 + c$

$$\Rightarrow c = 0$$

$$\text{Displacement } = \int_2^4 \left(3t + t^2\right) dt$$

$$= \left[\tfrac{3}{2}t^2 + \tfrac{1}{3}t^3\right]_2^4$$

$$= \left(24 + \tfrac{64}{3}\right) - \left(6 + \tfrac{8}{3}\right)$$

$$= \tfrac{110}{3}$$

The displacement is $\tfrac{110}{3}$ m.

10. (i) $s = t^3 - 3t^2 - 9t$

$$v = \frac{ds}{dt} = 3t^2 - 6t - 9$$

(ii) When $v = 0$, $3t^2 - 6t - 9 = 0$

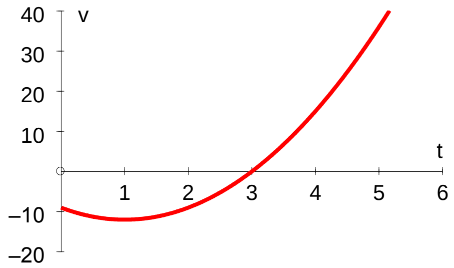
$$t^2 - 2t - 3 = 0$$

$$(t-3)(t+1) = 0$$

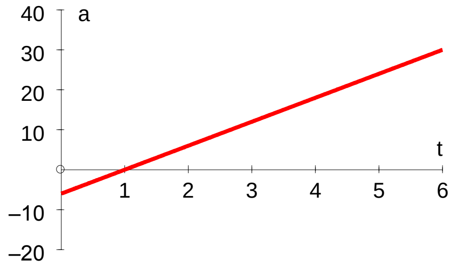
$$t = 3 \text{ or } t = -1$$

Since t must be positive, the body is at rest when $t = 3$.

(iii) $v = 3t^2 - 6t - 9$



$a = \frac{dv}{dt} = 6t - 6$



11. (i) $a = 2t - 6$

$v = \int (2t-6)\,dt = t^2 - 6t + c$

When $t = 0$, $v = 2$ so $2 = c$

$v = t^2 - 6t + 2$

$s = \int (t^2 - 6t + 2)\,dt = \frac{1}{3}t^3 - 3t^2 + 2t + k$

When $t = 0$, $s = 0 \Rightarrow k = 0$

$s = \frac{1}{3}t^3 - 3t^2 + 2t$

(ii) When $t = 5$, $v = 5^2 - 6 \times 5 + 2 = -3$

When $t = 5$, $s = \frac{1}{3} \times 5^3 - 3 \times 5^2 + 2 \times 5 = -\frac{70}{3}$

When t = 5, the velocity is -3 ms^{-2} and the displacement is $-\frac{70}{3}$ m.
This means that the particle has passed through O and is on the other side of O heading away from O.