Section 2: More about hypothesis testing

Solutions to Exercise level 2

1. Let p be the probability that a student obtains an A or B grade.

   $H_0 : p = 0.6$
   $H_1 : p < 0.6$

   Significance level = 5%

   Let X be the number of students who obtained an A or B grade. Need the highest possible value of a for which $P(X \le a) < 0.05$

   For B(19, 0.6), $P(X \le 7) = 0.0352$
   $P(X \le 8) = 0.0885$

   The highest possible value of a is 7.
   The critical region is $X \le 7$.

   The observed value of $X = 8$ does not lie in the critical region, so accept H_0.
   There is not sufficient evidence to support the Head of Department's concerns.

2. Let p be the probability of obtaining a head.

   $H_0 : p = 0.5$
   $H_1 : p > 0.5$

   Significance level = 5%

   Let X be the number of heads obtained.
   Need the lowest possible value of a for which $P(X \ge a) < 0.05$
   $\Rightarrow 1 - P(X \le a - 1) < 0.05$
   $\Rightarrow P(X \le a - 1) > 0.95$

   For B(18, 0.5), $P(X \le 11) = 0.8811$
   $P(X \le 12) = 0.9519$

   The lowest possible value of $a - 1$ is 12, so the lowest possible value of a is 13.
   The critical region is $X \ge 13$.

   The observed value of $X = 11$ does not lie in the critical region, so accept H_0.
   There is not sufficient evidence to suggest that the coin is biased.

3. Let p be the probability that a seed germinates.

$H_0 : p = 0.65$
$H_1 : p < 0.65$
Significance level = 5%

Let X be the number of seeds that germinate.
Need the highest possible value of a for which $P(X \leq a) < 0.05$

For B(16, 0.65), $P(X \leq 6) = 0.0229$
$P(X \leq 7) = 0.0671$

The highest possible value of a is 6.
The critical region is $X \leq 6$.

The observed value of $X = 8$ does not lie in the critical region, so accept H_0.
There is not sufficient evidence to suggest that there is any reduction in the germination rate.

4. Let p be the probability that a student does no fitness training or sporting activity out of school.

$H_0 : p = 0.7$
$H_1 : p < 0.7$

Significance level = 1%

Let X be the number of students who do no fitness training or sporting activity out of school.
Need the highest possible value of a for which $P(X \leq a) < 0.1$

For B(10, 0.7), $P(X \leq 4) = 0.0473$
$P(X \leq 5) = 0.1503$

The highest possible value of a is 4.
The critical region is $X \leq 4$.

The observed value of $X = 5$ does not lie in the critical region, so accept H_0.
There is not sufficient evidence to support the criticism by the sporting groups.

5. (i) Let p be the probability that a casualty has to wait more than 30 minutes
$H_0 : p = 0.3$
$H_1 : p < 0.3$

Let X be the number of patients who had to wait more than 30 minutes.
Using $X \sim B(20, 0.3)$ $\quad P(X \leq 2) = 0.0355$

At the 5% significance level:
$P(X \leq 2) < 0.05$, so reject H_0. There is evidence to suggest that the proportion of patients waiting more than 30 minutes has decreased.

(ii) At the 2% significance level:
$P(X \leq 2) > 0.02$, so accept H_0. There is not sufficient evidence to suggest that the proportion of patients waiting more than 30 minutes has decreased.

(iii) At the 5% level, need to find highest value of r such that $P(X \leq r) < 0.05$
From tables, $\quad P(X \leq 2) = 0.0355$
$\quad P(X \leq 3) = 0.1071$
The highest value of r is 2
The critical region is $X \leq 2$.

6. (i) Let p be the probability that a student gets a grade A – C.
$H_0 : p = \frac{2}{3}$
$H_1 : p \neq \frac{2}{3}$
Significance level = 5%

16 out of 20 is in the upper tail.
Let X be the number of students who got grades A - C.
Using $X \sim B(20, \frac{2}{3})$ $\quad P(X \geq 16) = 1 - P(X \leq 15)$

$$= 1 - 0.8485$$
$$= 0.1515$$

At the 5% significance level for a two-tailed test:
$P(X \geq 16) > 0.025$, so accept H_0. There is not sufficient evidence to suggest that the proportion of students getting grades A – C is different.

(ii) Let p be the probability that a student gets a grade A – C.
$H_0 : p = \frac{2}{3}$
$H_1 : p > \frac{2}{3}$
Significance level = 5%

Let X be the number of students who got grades A - C.
Using $X \sim B(20, \frac{2}{3})$ $\quad P(X \geq 16) = 1 - P(X \leq 15)$

$$= 1 - 0.8485$$
$$= 0.1515$$

At the 5% significance level for a one-tailed test:

$P(X \geq 16) > 0.05$, so accept H_0. There is not sufficient evidence to suggest that the proportion of students getting grades A – C has increased.

7. (i) Let p be the probability that a pass is missed.
$H_0: p = 0.6$
$H_1: p < 0.6$
Significance level = 5%

Let X be the number of passes which are missed.
Using $X \sim B(17, 0.6)$ $\quad P(X \leq 5) = 0.0106$

At the 5% significance level for a one-tailed test:
$P(X \leq 5) < 0.05$, so reject H_0. There is evidence to suggest that the proportion of missed passes has decreased.

(ii) Let p be the probability that a pass is missed.
$H_0: p = 0.6$
$H_1: p \neq 0.6$
Significance level = 5%

Let X be the number of passes which are missed.
Using $X \sim B(17, 0.6)$ $\quad P(X \leq 5) = 0.0106$

At the 5% significance level for a two-tailed test:
$P(X \leq 5) < 0.025$, so reject H_0. There is evidence to suggest that the proportion of missed passes has decreased.