Section 2: More about hypothesis testing

Solutions to Exercise level 1

1. (i) $H_0 : p = \frac{1}{6}$

$H_1 : p < \frac{1}{6}$

(ii) Let X be the number of sixes in 20 throws

$X \sim B(30, \frac{1}{6})$

$P(X \leq 1) = 0.029$

$P(X \leq 2) = 0.103$

(iii) So the critical region is $X \leq 1$. This means that if Jessica gets 1 or fewer sixes in her 30 throws, she will conclude that there is evidence that the dice is biased against six.

2. (i) $H_0 : p = \frac{1}{6}$

$H_1 : p > \frac{1}{6}$

(ii) Let X be the number of ones in 12 throws

$X \sim B(50, \frac{1}{6})$

$P(X \geq 12) = 1 - P(X \leq 11)$

$= 1 - 0.883$

$= 0.117$

$P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - 0.937$

$= 0.063$

(iii) So the critical region is $X \geq 13$. This means that if Hassan gets 13 or more ones, he will conclude that there is evidence that the dice is biased towards one.

3. (i) $H_0 : p = \frac{1}{2}$

$H_1 : p \neq \frac{1}{2}$

(ii) $X \sim B(50, \frac{1}{2})$

$P(X \leq 18) = 0.032$

Since this is a two-tailed test, the p-value $= 2 \times 0.032 = 0.064$

(iii) p-value > 0.05, so accept H_0. There is not sufficient evidence to suggest that the coin is biased.

(iv) Each tail needs to have a probability of less than 2.5%

$P(X \leq 18) = 0.032$

$P(X \leq 17) = 0.016$

By symmetry $P(X \geq 32) = 0.032$ and $P(X \geq 33) = 0.016$

The critical region is $X \leq 17$ and $X \geq 33$.