Section 1: Introducing hypothesis testing

Solutions to Exercise level 2

1. Let p be the probability that a student gets an A or B grade.

   $H_0 : p = 0.4$
   $H_1 : p < 0.4$

   Significance level = 5%

   Let X be the number of students who get an A or B grade
   For $X \sim B(19, 0.4)$, $P(X \leq 2) = 0.0055 < 0.05$
   Reject H_0. There is evidence to suggest that the A and B pass-rate has decreased.

2. Let p be the probability of obtaining a head

   $H_0 : p = 0.5$
   $H_1 : p > 0.5$

   Significance level = 5%

   Let X be the number of heads obtained
   For $X \sim B(8, 0.5)$, $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.8555 = 0.1445 > 0.05$
   Accept H_0. There is not sufficient evidence to suggest that the coin is biased towards heads.

3. Let p be the probability of obtaining a head

   $H_0 : p = 0.5$
   $H_1 : p > 0.5$

   Significance level = 5%

   Let X be the number of heads obtained
   For $X \sim B(16, 0.5)$, $P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9616 = 0.0384 < 0.05$
   Reject H_0. There is evidence to suggest that the coin is biased towards heads.

4. Let p be the probability that a seed germinates

   $H_0 : p = 0.75$
   $H_1 : p < 0.75$

Significance level = 5%

Let X be the number of seeds which germinate
For $X \sim B(16, 0.75)$, $P(X \leq 8) = 0.0271 < 0.05$
Reject H_0. There is evidence to suggest that the new batch has a lower germination rate.

5. Let p be the probability that a passenger loses his suitcase

$H_0 : p = 0.05$
$H_1 : p > 0.05$

Significance level = 5%.

Let X be the number of times a passenger loses his suitcase
For $X \sim B(25, 0.05)$:

$$P(X \geq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$$
$$= 1 - (0.95)^{25} - 25(0.05)(0.95)^{24} - \frac{25 \times 24}{1 \times 2}(0.05)^2(0.95)^{23}$$
$$= 0.1271$$
$$> 0.05$$

Accept H_0. There is not sufficient evidence to suggest that the true probability is greater than 0.05.

6. (i) $H_0 : p = 0.8$
$H_1 : p > 0.8$

(ii) Let X be the number of times the bus is late in 10 journeys
$X \sim B(10, 0.8)$
$P(X \geq 10) = P(X = 10)$
$= 0.8^{10}$
$= 0.107$
The p-value is 0.107

(iii) 0.141 > 0.1, so accept H_0. There is insufficient evidence that the bus is on time more than 80% of the time.

(iv) The null hypothesis was accepted even though the bus was on time on every occasion, so it is not possible with this number of journeys to conclude that the service has improved.
The minimum number of journeys should be 14 since $0.8^{14} < 0.05$, so it would then be possible to reject H_0 if the bus is on time on every occasion.

