Section 1: Introducing the binomial distribution

Solutions to Exercise level 3

1. (i) The number of matches in a box is independent of the numbers of matches in other boxes
The probability that a matchbox contains fewer than 42 matches is constant.

(ii) $Y \sim B(8, 0.1)$
$P(Y = 0) = 0.9^8 = 0.4305$ (4 s.f.)
$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.8131 = 0.1869$
Probability $= 2 \times 0.4305 \times 0.1869 = 0.161$ (3 s.f.)

2. (i) Let X be the number of seeds that germinate, so $X \sim B(10, 0.9)$
Expected number to germinate is 9
$P(X < 9) = P(X \leq 8) = 0.2639$ (4 s.f.)

(ii) $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.0702 = 0.9298$ (4 s.f.)
Let Y be the number of trays in which at least 8 seeds germinate,
so $Y \sim B(20, 0.9298)$
$P(Y \geq 19) = P(Y = 19) + P(Y = 20)$
$= 20 \times 0.9298^{19} \times 0.0702 + 0.9298^{20}$
$= 0.5854$ (4 s.f.)

3. $W \sim B(n, 0.27)$
$P(W \geq 1) > 0.95$
$1 - P(W = 0) > 0.95$
$P(W = 0) < 0.05$
$0.73^n < 0.05$
$n \log 0.73 < \log 0.05$
$n > 9.5$
so the smallest value of n is 10.

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4. $T \sim B(11,p)$

$\text{Var}(T) = 1.76 \Rightarrow 11p(1-p) = 1.76$

$\Rightarrow p(1-p) = 0.16$

$\Rightarrow 25p - 25p^2 = 4$

$\Rightarrow 25p^2 - 25p + 4 = 0$

$\Rightarrow (5p-4)(5p-1) = 0$

$\Rightarrow p = 0.8 \text{ or } 0.2$

5. (i) Let number of times waiting more than six minutes be X, so $X \sim B(10, 0.4)$

(a) $P(X \leq 3) = 0.3823$ (4 s.f)

(b) $P(X > 4) = 1 - P(X \leq 4) = 1 - 0.6331 = 0.3669$ (4 s.f.)

(ii) $\text{Mean} = np = 10 \times 0.4 = 4$

$\text{Standard deviation} = \sqrt{npq} = \sqrt{10 \times 0.4 \times 0.6} = 1.549$ (4 s.f.)

(iii) (a) $\text{Mean} = \frac{51}{13} = 3.923$ (4 s.f.)

$$\text{Standard deviation} = \sqrt{\frac{321 - 13\left(\frac{51}{13}\right)^2}{12}} = 3.174 \text{ (4 s.f)}$$

(b) The data do not support Claire's beliefs, as although the theoretical mean and the sample mean are close (3.9 is close to 4), there is quite a large difference between the theoretical standard deviation of 1.5 and the sample standard deviation of 3.2. This suggests that a binomial model is not appropriate.

