Section 1: Introducing the binomial distribution

Solutions to Exercise level 2

1. Let X be the number of correct answers, so $X \sim B(10, 0.2)$
   (i) $P(X = 1) = {}_{10}C_1 \times 0.2^1 \times 0.8^9 = 0.2684$ (4 s.f.)
   (ii) $P(X = 5) = {}_{10}C_5 \times 0.2^5 \times 0.8^5 = 0.02642$ (4 s.f.)
   (iii) $P(X < 3) = P(X \le 2) = 0.6778$ (4 s.f.)
   (iv) $P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.3758 = 0.6242$ (4 s.f.)

2. Let X be the number of heads obtained, so $X \sim B(10, 0.4)$
   (i) $P(X < 5) = P(X \le 4) = 0.6331$ (4 s.f.)
   (ii) $P(X = 5) = {}_{10}C_5 \times 0.4^5 \times 0.6^5 = 0.2007$ (4 s.f.)
   (iii) $P(X > 3) = 1 - P(X \le 3) = 1 - 0.3823 = 0.6177$ (4 s.f.)
   (iv) Let Y be the number of occasions on which exactly 5 heads are obtained, so $Y \sim B(7, 0.2007)$
   $P(Y = 2) = {}_7C_2 \times 0.2007^2 \times 0.7993^5 = 0.276$ (3 s.f.)

3. Let X be the number of white bulbs.
   $X \sim B(n, 0.2)$
   $P(X \ge 1) > 0.95$
   $1 - P(X = 0) > 0.95$
   $P(X = 0) < 0.05$
   $0.8^n < 0.05$
   $0.8^{13} = 0.055$ and $0.8^{14} = 0.044$
   The least number of bulbs that must be selected is 14.

4. (i) $X \sim B(6, 0.15)$
   (a) $P(X = 0) = 0.85^6 = 0.377$ (3 s.f.)
   (b) $P(X = 1) = 6 \times 0.15 \times (0.85)^5 = 0.399$ (3 s.f.)
   (c) $P(X > 1) = 1 - P(X \le 1)$
   $= 1 - 0.776$
   $= 0.224$ (3 s.f.)

(d) $P(X=3) = {}_6C_3(0.15)^3(0.85)^3$

$$= \frac{6\times5\times4}{1\times2\times3}(0.15)^3(0.85)^3 = 0.0415 \text{ (3 s.f.)}$$

(ii) Let Y be the number of weeks in which I arrive with my suitcase on all flights.

$Y \sim B(4, 0.85^6)$

$P(Y=3) = 4\times(0.85^6)^3\times(1-0.85^6) = 0.134$

5. (i) Could be argued either way – either that it is reasonable to assume that the each trial is independent and the probability of success is constant, or that it is not reasonable as the outcome of each trial could affect the next (improving with practice, or loss of confidence).

(ii) Let X be the number of scores in 10 free shots, so $X \sim B(10, 0.35)$

$P(X<4) = P(X\le 3) = 0.5138$ (4 s.f.)

(iii) Let Y be the number of times that she scores fewer than 4 times in a set, so $Y \sim B(5, 0.5138)$

$P(Y=3) = {}_5C_3 \times 0.5138^3 \times 0.4862^2 = 0.321$ (3 s.f.)

6. $P(X=10) = {}_{21}C_{10}\times p^{10}(1-p)^{11}$

$P(X=9) = {}_{21}C_9\times p^9(1-p)^{12}$

$${}_{21}C_{10}\times p^{10}(1-p)^{11} = {}_{21}C_9\times p^9(1-p)^{12}$$

$$\frac{21!}{10!11!}p^{10}(1-p)^{11} = \frac{21!}{9!12!}p^9(1-p)^{12}$$

$$\frac{12!}{11!}p = \frac{10!}{9!}(1-p)$$

$$12p = 10 - 10p$$

$$22p = 10$$

$$p = \tfrac{5}{11}$$

7. (i) Let X be the number of left-handed people in a sample of 10, so $X \sim B(10, 0.2)$

$P(X=3) = {}_{10}C_3\times 0.2^3\times 0.8^7 = 0.2013$ (4 s.f.)

(ii) Let Y be the number of left-handed people in a sample of 15, so $Y \sim B(15, 0.2)$

$P(Y>7.5) = 1 - P(Y\le 7) = 1 - 0.9958 = 0.0042$ (4 s.f.)

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(iii) Let Z be the number of left-handed people in a sample of 12, so $Z \sim B(12, 0.2)$

Mean of $Z = 12 \times 0.2 = 2.4$

$P(Z = 2) = {}_{12}C_2 \times 0.2^2 \times 0.8^{10} = 0.2835$ (4 s.f.)

$P(Z = 3) = {}_{12}C_3 \times 0.2^3 \times 0.8^9 = 0.2262$ (4 s.f.)

so the most likely number is 2.

(iv) Let $W \sim B(n, 0.2)$

$P(W = 0) = 0.8^n$

$0.8^n < 0.05$

$n \log 0.8 < \log 0.05$

$n > 13.4$

so the sample must be at least 14 people.