Section 1: Introducing the binomial distribution

Solutions to Exercise level 1

1. $X \sim B(8, 0.6)$

(i) $P(X = 0) = (0.4)^8 = 0.000655$ (3 s.f.)

(ii) $P(X = 3) = {}_8C_3(0.6)^3(0.4)^5 = \frac{8 \times 7 \times 6}{1 \times 2 \times 3}(0.6)^3(0.4)^5 = 0.124$ (3 s.f.)

(iii) $P(X = 6) = {}_8C_6(0.6)^6(0.4)^2 = \frac{8 \times 7}{1 \times 2}(0.6)^6(0.4)^2 = 0.209$ (3 s.f.)

2. $X \sim B(10, 0.7)$

(i) $P(X = 0) = (0.3)^{10} = 0.00000590$ (3 s.f.)

(ii) $P(X = 1) = {}_{10}C_1 \times 0.7 \times (0.3)^9 = 10 \times 0.7 \times (0.3)^9 = 0.000138$ (3 s.f.)

(iii) $P(X > 1) = 1 - P(X \leq 1)$
$= 1 - 0.000143$
$= 0.9999$ (4 s.f.)

(iv) $P(X = 2) = {}_{10}C_2(0.7)^2(0.3)^8 = \frac{10 \times 9}{1 \times 2}(0.7)^2(0.3)^8 = 0.001447$ (4 s.f.)

$P(X < 3) = P(X \leq 2)$
$= 0.00159$ (3 s.f.)

3. $X \sim B(12, 0.4)$

$P(X = 4) = {}_{12}C_4(0.4)^4(0.6)^8$
$= \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4}(0.4)^4(0.6)^8 = 0.213$

$P(X = 5) = {}_{12}C_5(0.4)^5(0.6)^7$
$= \frac{12 \times 11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4 \times 5}(0.4)^5(0.6)^7 = 0.227$

$P(X = 6) = {}_{12}C_6(0.4)^6(0.6)^6$
$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4 \times 5 \times 6}(0.4)^6(0.6)^6 = 0.177$

The most likely outcome for X is 5.

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4. $X \sim B(8, 0.4)$

$P(X = 3) = {}_8C_3(0.4)^3(0.6)^5$

$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3}(0.4)^3(0.6)^5 = 0.279$

$P(X = 4) = {}_8C_4(0.4)^4(0.6)^4$

$= \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}(0.4)^4(0.6)^4 = 0.232$

The most likely outcome for X is 3.

5. Let X be the number of students who pass

$X \sim B(5, 0.9)$

(i) $P(X = 5) = (0.9)^5 = 0.590$ (3 s.f.)

(ii) $P(X = 2) = {}_5C_2 \times (0.9)^2 \times (0.1)^3 = \frac{5 \times 4}{1 \times 2} \times (0.9)^2 \times (0.1)^3 = 0.0081$

(iii) $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.00856$

$= 0.991$ (3 s.f.)

(iv) Since the probability that all five students pass is more than 0.5, this must be the greatest probability.
So the most likely number of students who pass is 5.

6. Let X be the number of times I lose my suitcase

$X \sim B(6, 0.05)$

(i) $P(X = 0) = (0.95)^6 = 0.735$ (3 s.f.)

(ii) $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.735 = 0.265$ (3 s.f.)

(iii) $P(X = 1) = {}_6C_1(0.05)(0.95)^5 = 6(0.05)(0.95)^5 = 0.232$ (3 s.f.)

7. Let X be the number of orders dispatched on the next working day.

$X \sim B(10, 0.75)$

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(i) $P(X = 4) = {}_{10}C_4(0.75)^4(0.25)^6$

$= \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}(0.75)^4(0.25)^6$

$= 0.0162$ (3 s.f.)

(ii) $P(X < 4) = P(X \leq 3)$

$= 0.00351$ (3 s.f.)