Section 4: Finding distances

Exercise level 1

1. (i) Distance $= \left|\dfrac{(2\times1)+(1\times-2)+(-4\times1)+22}{\sqrt{1^2+2^2+1^2}}\right| = \dfrac{18}{\sqrt{6}} = 3\sqrt{6}$

(ii) L has equation $\underset{\sim}{r} = \begin{pmatrix}2\\1\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}$

(ii) For a point on L, $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2+\lambda\\1-2\lambda\\-4+\lambda\end{pmatrix}$

Substituting into the equation of the plane:

$(2+\lambda)-2(1-2\lambda)+(-4+\lambda)+22=0$

$6\lambda = -18$

$\lambda = -3$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2-3\\1+6\\-4-3\end{pmatrix} = \begin{pmatrix}-1\\7\\-7\end{pmatrix}$

The coordinates of Q are (-1, 7, -7)

(iv) The distance of P (2, 1, -4) to Q (-1, 7, -7) is given by

$PQ^2 = (2-(-1))^2+(1-7)^2+(-4-(-7))^2$

$= 9+36+9$

$= 54$

$PQ = \sqrt{54} = 3\sqrt{6}$

2. (i) $\overrightarrow{OM} = \begin{pmatrix}1-2\lambda\\3\lambda\\3+\lambda\end{pmatrix}$

$\overrightarrow{PM} = \begin{pmatrix}1-2\lambda\\3\lambda\\3+\lambda\end{pmatrix} - \begin{pmatrix}-5\\5\\4\end{pmatrix} = \begin{pmatrix}6-2\lambda\\3\lambda-5\\\lambda-1\end{pmatrix}$

(ii) $\begin{pmatrix} 6-2\lambda \\ 3\lambda-5 \\ \lambda-1 \end{pmatrix} . \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = 0$

$-2(6-2\lambda)+3(3\lambda-5)+(\lambda-1)=0$

$-12+4\lambda+9\lambda-15+\lambda-1=0$

$14\lambda=28$

$\lambda=2$

(iii) $\lambda=2$ so $\overrightarrow{OM}=\begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix}$

M = (-3, 6, 5)

(iv) $\overrightarrow{PM}=\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\left|\overrightarrow{PM}\right|=\sqrt{2^2+1^2+1^2}=\sqrt{6}$

3. (i) $r=\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}+\lambda\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$

(ii) Let M be the point on line L_1 which is closest to (2, 3, -1).

$\overrightarrow{OM}=\begin{pmatrix} 10+5\lambda \\ 3+2\lambda \\ -13-3\lambda \end{pmatrix}$

$\overrightarrow{PM}=\begin{pmatrix} 10+5\lambda \\ 3+2\lambda \\ -13-3\lambda \end{pmatrix}-\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}=\begin{pmatrix} 8+5\lambda \\ 2\lambda \\ -12-3\lambda \end{pmatrix}$

$\overrightarrow{PM}$ is perpendicular to L_1 so $\begin{pmatrix} 8+5\lambda \\ 2\lambda \\ -12-3\lambda \end{pmatrix} . \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} = 0$

$5(8+5\lambda)+2(2\lambda)-3(-12-3\lambda)=0$

$38\lambda=-76$

$\lambda=-2$

$$\overrightarrow{PM} = \begin{pmatrix} -2 \\ -4 \\ -6 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\left|\overrightarrow{PM}\right| = 2\sqrt{1^2 + 2^2 + 3^2} = 2\sqrt{14}$$

4. (i) $\overrightarrow{OP} = \begin{pmatrix} -8 + 2\lambda \\ -13 - 3\lambda \\ 28 + \lambda \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 11 + \mu \\ -4 + 4\mu \\ -15 + 2\mu \end{pmatrix}$

$$\overrightarrow{PQ} = \begin{pmatrix} 11 + \mu \\ -4 + 4\mu \\ -15 + 2\mu \end{pmatrix} - \begin{pmatrix} -8 + 2\lambda \\ -13 - 3\lambda \\ 28 + \lambda \end{pmatrix} = \begin{pmatrix} 19 + \mu - 2\lambda \\ 9 + 4\mu + 3\lambda \\ -43 + 2\mu - \lambda \end{pmatrix}$$

(ii) $$\begin{pmatrix} 19 + \mu - 2\lambda \\ 9 + 4\mu + 3\lambda \\ -43 + 2\mu - \lambda \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$$

$$2(19 + \mu - 2\lambda) - 3(9 + 4\mu + 3\lambda) + (-43 + 2\mu - \lambda) = 0$$

$$-14\lambda - 8\mu - 32 = 0$$

$$7\lambda + 4\mu = -16$$

(iii) $$\begin{pmatrix} 19 + \mu - 2\lambda \\ 9 + 4\mu + 3\lambda \\ -25 + 2\mu - \lambda \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$(19 + \mu - 2\lambda) + 4(9 + 4\mu + 3\lambda) + 2(-43 + 2\mu - \lambda) = 0$$

$$8\lambda + 21\mu = 31$$

(iv) (1) $7\lambda + 4\mu = -16$

(2) $8\lambda + 21\mu = 31$

(1)×8 $56\lambda + 32\mu = -128$

(2)×7 $56\lambda + 147\mu = 217$

Subtracting: $115\mu = 345$

$$\mu = 3, \lambda = -4$$

(v) P = (-16, -1, 24), Q = (14, 8, -9)

$$\overrightarrow{PQ} = \begin{pmatrix} 30 \\ 9 \\ -33 \end{pmatrix} = 3\begin{pmatrix} 10 \\ 3 \\ -11 \end{pmatrix}$$

$$\left|\overrightarrow{PQ}\right| = 3\sqrt{10^2 + 3^2 + 11^2} = 3\sqrt{230}$$