

## Section 1: The scalar product

## Solutions to Exercise level 1

$$1. \quad (i) \quad \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -5 \end{pmatrix} = (3 \times 4) + (1 \times -5) = 12 - 5 = 7$$

$$(ii) \quad (2\hat{i} + 3\hat{j}) \cdot (-\hat{i} + 2\hat{j}) = (2 \times -1) + (3 \times 2) = -2 + 6 = 4$$

$$2. \quad (i) \quad \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix} = (1 \times 3) + (-4 \times 2) + (2 \times -5) \\ = 3 - 8 - 10 \\ = -15$$

$$(ii) \quad (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{i} - 2\hat{k}) = (1 \times 4) + (2 \times 0) + (3 \times -2) \\ = 4 + 0 - 6 \\ = -2$$

3. The vectors are perpendicular so the scalar product is 0.

$$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -2 \\ k \end{pmatrix} = 0 \\ (3 \times 4) + (2 \times -2) + (-1 \times k) = 0 \\ 12 - 4 - k = 0 \\ k = 8$$

$$4. \quad (i) \quad (3\hat{i} - 5\hat{j}) \cdot (2\hat{i} + \hat{j}) = (3 \times 2) + (-5 \times 1) = 6 - 5 = 1$$

$$|3\hat{i} - 5\hat{j}| = \sqrt{3^2 + (-5)^2} = \sqrt{34}$$

$$|2\hat{i} + \hat{j}| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\cos \theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} = \frac{1}{\sqrt{34} \sqrt{5}}$$

$$\theta = 85.6^\circ \text{ (1 d.p.)}$$

$$(ii) \quad (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k}) = (1 \times 2) + (-2 \times 1) + (3 \times -1) = 2 - 2 - 3 = -3$$

## Edexcel FM Vectors 1 Exercise solutions

$$|\underline{i} - 2\underline{j} + 3\underline{k}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$|2\underline{i} + \underline{j} - \underline{k}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$$

$$\cos \theta = \frac{-3}{\sqrt{14}\sqrt{6}}$$

$$\theta = 109.1^\circ$$

$$5. \quad (i) \quad \overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 8 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix}$$

$$(ii) \quad \overrightarrow{AB} \cdot \overrightarrow{AC} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} \\ = (-1 \times -1) + (1 \times 5) + (1 \times -2) \\ = 1 + 5 - 2 \\ = 4$$

$$(ii) \quad |\overrightarrow{AB}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3} \\ |\overrightarrow{AC}| = \sqrt{1^2 + (-5)^2 + 2^2} = \sqrt{30} \\ \cos \theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| |\overrightarrow{AC}|} \\ = \frac{4}{\sqrt{3}\sqrt{30}} \\ \theta = 65.1^\circ$$