Section 2: The vector equation of a line

Exercise level 3 solutions

1. Let the direction vector of these line be $\begin{pmatrix} p \\ q \end{pmatrix}$.

This vector makes an angle of 30° with the vector $\begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix}$.

$$\text{So } \cos 30° = \frac{\begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} p \\ q \end{pmatrix}\right| \left|\begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix}\right|} = \frac{p\sqrt{3} + 3q}{\sqrt{(p^2 + q^2)(3 + 3^2)}}$$

$$\frac{\sqrt{3}}{2} = \frac{p\sqrt{3} + 3q}{2\sqrt{3(p^2 + q^2)}}$$

$$3\sqrt{p^2 + q^2} = p\sqrt{3} + 3q$$

$$9p^2 + 9q^2 = 3p^2 + 6pq\sqrt{3} + 9q^2$$

$$6p^2 = 6pq\sqrt{3}$$

$$p(p - q\sqrt{3}) = 0$$

So either $p = 0$ or $p = q\sqrt{3}$

so the direction vectors of the new lines are $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$

Both lines go through (0, 2)

So possible equations for the lines are

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \underset{\sim}{r} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$$