Section 2: The vector equation of a line

Solutions to Exercise level 1

1. (i) Direction vector is $\begin{pmatrix}3\\-1\end{pmatrix}-\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}1\\-6\end{pmatrix}$

Equation of line is $\underset{\sim}{r}=\begin{pmatrix}2\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\-6\end{pmatrix}$

(ii) Direction vector is $\begin{pmatrix}1\\6\end{pmatrix}-\begin{pmatrix}-3\\2\end{pmatrix}=\begin{pmatrix}4\\4\end{pmatrix}=4\begin{pmatrix}1\\1\end{pmatrix}$

Equation of line is $\underset{\sim}{r}=\begin{pmatrix}-3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\end{pmatrix}$

(iii) Direction vector is $\begin{pmatrix}3\\-1\end{pmatrix}$

Equation of line is $\underset{\sim}{r}=\begin{pmatrix}0\\6\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\end{pmatrix}$

2. (i) $\begin{pmatrix}3\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}+\mu\begin{pmatrix}2\\2\end{pmatrix}$

$3+\lambda=1+2\mu$

$3-\lambda=2\mu$

Adding: $6=1+4\mu$

$5=4\mu$

$\mu=\frac{5}{4}$

Point of intersection $=\begin{pmatrix}1\\0\end{pmatrix}+\frac{5}{4}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}3.5\\2.5\end{pmatrix}$

The coordinates of the point of intersection are (3.5, 2.5)

(ii) $\begin{pmatrix}2\\1\end{pmatrix}+\lambda\begin{pmatrix}3\\0\end{pmatrix}=\begin{pmatrix}4\\3\end{pmatrix}+\mu\begin{pmatrix}1\\2\end{pmatrix}$

$2+3\lambda=4+\mu$

$1=3+2\mu \quad \Rightarrow -2=2\mu \quad \Rightarrow -1=\mu$

Point of intersection is $\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$

The coordinates of the point of intersection are (3, 1)

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3. (i) Angle between lines is angle between direction vectors $3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $2\underset{\sim}{i} - 3\underset{\sim}{j}$.

$$(3\underset{\sim}{i} - 4\underset{\sim}{j}).(2\underset{\sim}{i} - 3\underset{\sim}{j}) = (3\times 2) + (-4\times -3) = 18$$

$$\left|3\underset{\sim}{i} - 4\underset{\sim}{j}\right| = \sqrt{3^2 + 4^2} = 5$$

$$\left|2\underset{\sim}{i} - 3\underset{\sim}{j}\right| = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\cos\theta = \frac{18}{5\sqrt{13}}$$

$$\theta = 3.2^\circ \text{ (1 d.p.)}$$

(ii) (a) $\left(8\underset{\sim}{i} + 6\underset{\sim}{j}\right)\left(3\underset{\sim}{i} - 4\underset{\sim}{j}\right) = (8\times 3) + (6\times -4) = 24 - 24 = 0$

so the line $\underset{\sim}{r} = 2\underset{\sim}{i} - \underset{\sim}{j} + \lambda(3\underset{\sim}{i} - 4\underset{\sim}{j})$ is perpendicular to $8\underset{\sim}{i} + 6\underset{\sim}{j}$.

(b) $\left(6\underset{\sim}{i} + 4\underset{\sim}{j}\right)\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right) = (6\times 2) + (4\times -3) = 12 - 12 = 0$

so the line $\underset{\sim}{s} = 3\underset{\sim}{i} + \underset{\sim}{j} + \mu(2\underset{\sim}{i} - 3\underset{\sim}{j})$ is perpendicular to $6\underset{\sim}{i} + 4\underset{\sim}{j}$.

(iii) The line $\underset{\sim}{r} = 2\underset{\sim}{i} - \underset{\sim}{j} + \lambda(3\underset{\sim}{i} - 4\underset{\sim}{j})$ is parallel to the direction vector $3\underset{\sim}{i} - 4\underset{\sim}{j}$.

The unit vector is $\dfrac{3\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{3^2 + 4^2}} = \dfrac{3}{5}\underset{\sim}{i} - \dfrac{4}{5}\underset{\sim}{j}$

The line $\underset{\sim}{s} = 3\underset{\sim}{i} + \underset{\sim}{j} + \mu(2\underset{\sim}{i} - 3\underset{\sim}{j})$ is parallel to the direction vector $2\underset{\sim}{i} - 3\underset{\sim}{j}$.

The unit vector is $\dfrac{2\underset{\sim}{i} - 3\underset{\sim}{j}}{\sqrt{2^2 + 3^2}} = \dfrac{2}{\sqrt{13}}\underset{\sim}{i} - \dfrac{3}{\sqrt{13}}\underset{\sim}{j}$

4. Direction vector $= \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix}$

Alternatively the negative of this vector could be used

Vector equation of line is $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix}$

The position vector of the point (-2, 3, 5) could be used instead of (3, 1, 1)

Cartesian equation of line is $\dfrac{x-3}{5} = \dfrac{y-1}{-2} = \dfrac{z-1}{-4}$

5. $\dfrac{x-3}{-1}=\dfrac{y-1}{2}=\dfrac{z-3}{2}$

6. $r=\begin{pmatrix}1\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\2\end{pmatrix}$

7. (i) $\underset{\sim}{r}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-3\end{pmatrix}$ and $\underset{\sim}{r}=\begin{pmatrix}5\\3\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\4\end{pmatrix}$

At intersection, $\begin{cases}-2+2\lambda=5+\mu & (1)\\ 1=3+2\mu & (2)\\ 3-3\lambda=-2+4\mu & (3)\end{cases}$

(2) $\Rightarrow 2\mu=-2 \quad \Rightarrow \mu=-1$

Substituting into (1): $\quad 2\lambda=7+\mu=6 \quad \Rightarrow \lambda=3$

For (3): $\quad$ LHS $=3-9=-6$, RHS $=-2-4=-6$

So the lines do intersect.

Position vector of intersection point is $\underset{\sim}{r}=\begin{pmatrix}5\\3\\-2\end{pmatrix}-\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}4\\1\\-6\end{pmatrix}$

Intersection point is (4, 1, -6).

(ii) $\underset{\sim}{r}=\begin{pmatrix}3\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-3\end{pmatrix}$ and $\underset{\sim}{r}=\begin{pmatrix}5\\1\\-3\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\1\end{pmatrix}$

At intersection, $\begin{cases}3+\lambda=5-2\mu & (1)\\ -1+2\lambda=1+\mu & (2)\\ -3\lambda=-3+\mu & (3)\end{cases}$

(2) – (3): $\quad -1+5\lambda=4 \quad \Rightarrow \lambda=1,\ \mu=0$

For (1): LHS $=3+1=4$, RHS $=5-0=5$

so the lines do not intersect.

(iii) $\underset{\sim}{r}=\begin{pmatrix}2\\6\\-3\end{pmatrix}+\lambda\begin{pmatrix}4\\2\\-3\end{pmatrix}$ and $\underset{\sim}{r}=\begin{pmatrix}2\\0\\4\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-3\end{pmatrix}$

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At intersection, $\begin{cases} 2+4\lambda = 2+\mu & (1) \\ 6+2\lambda = -\mu & (2) \\ -3-3\lambda = 4-3\mu & (3) \end{cases}$

(1) + (2): $8+6\lambda = 2 \quad \Rightarrow \lambda = -1,\ \mu = -4$

For (3): LHS $= -3+3 = 0$, RHS $= 4+12 = 16$

so the lines do not intersect.

(iv) $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -5 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

At intersection, $\begin{cases} 1-\lambda = 1+2\mu & (1) \\ 2+2\lambda = -5+3\mu & (2) \\ 6+3\lambda = 4-4\mu & (3) \end{cases}$

2× (1) + (2): $4 = -3+7\mu \quad \Rightarrow \mu = 1,\ \lambda = -2$

For (3): LHS $= 6-6 = 0$, RHS $= 4-4 = 0$

so the lines do intersect.

Position vector of intersection point is $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -5 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$

The intersection point is (3, -2, 0).