# Section 2: Proof by induction

## Solutions to Exercise level 1

1. (i) $u_1 = 3 - 2^{1-1} = 3 - 1 = 2$
   So the result is true for $n = 1$.

   (ii) $u_{k+1} = 2u_k - 3$
   $= 2(3 - 2^{k-1}) - 3$

   (iii) $u_{k+1} = 2\times 3 - 2\times 2^{k-1} - 3$
   $= 6 - 2^k - 3$
   $= 3 - 2^k$

   (iv) So if the result is true for $n = k$, then it is true for $n = k + 1$.
   Since it is true for $n = 1$, then it is true for all positive integers greater than or equal to 1 by induction.

2. (i) When $n = 1$, $A^1 = \begin{pmatrix} 2^1 & 0 \\ 2^2 - 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 4-2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$.
   So the result is true for $n = 1$.

   (ii) Assume that $A^k = \begin{pmatrix} 2^k & 0 \\ 2^{k+1} - 2 & 1 \end{pmatrix}$

   $$A^k A = \begin{pmatrix} 2^k & 0 \\ 2^{k+1} - 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$$
   $$= \begin{pmatrix} 2\times 2^k & 0 \\ 2\times 2^{k+1} - 2\times 2 + 2 & 1 \end{pmatrix}$$

   (iii) $$A^{k+1} = \begin{pmatrix} 2\times 2^k & 0 \\ 2\times 2^{k+1} - 2\times 2 + 2 & 1 \end{pmatrix}$$
   $$= \begin{pmatrix} 2^{k+1} & 0 \\ 2^{k+2} - 4 + 2 & 1 \end{pmatrix}$$
   $$= \begin{pmatrix} 2^{k+1} & 0 \\ 2^{k+2} - 2 & 1 \end{pmatrix}$$

   (iv) So if the result is true for $n = k$, then it is true for $n = k + 1$.
   Since it is true for $n = 1$, then it is true for all positive integers greater

 

than or equal to 1 by induction.

3. (i) When $n = 1$, L.H.S. $= 1$

$$\text{R.H.S} = \tfrac{1}{2}\times 1(3-1) = \tfrac{1}{2}\times 1\times 2 = 1$$

So the result is true for $n = 1$.

(ii) The (k+1)th term $= 3(k+1)-2$

(iii) Assume sum of k terms $= \tfrac{1}{2}k(3k-1)$

Sum of (k+1) terms $= \tfrac{1}{2}k(3k-1)+3(k+1)-2$

(iv)
$$\begin{aligned}\tfrac{1}{2}k(3k-1)+3(k+1)-2 &= \tfrac{3}{2}k^2-\tfrac{1}{2}k+3k+3-2\\ &= \tfrac{3}{2}k^2+\tfrac{5}{2}k+1\\ &= \tfrac{1}{2}(3k^2+5k+2)\\ &= \tfrac{1}{2}(k+1)(3k+2)\\ &= \tfrac{1}{2}(k+1)(3(k+1)-1)\end{aligned}$$

(v) So if the result is true for $n = k$, then it is true for $n = k + 1$.
Since it is true for $n = 1$, then it is true for all positive integers greater than or equal to 1 by induction.

4. (i) When $n = 1$, L.H.S. $= 2-3 = -1$

$$\text{R.H.S} = 1(1-2) = 1\times -1 = -1$$

So the result is true for $n = 1$.

(ii) (k+1)th term $= 2(k+1)-3$

(iii) Assume $\sum_{r=1}^{k}(2r-3) = k(k-2)$

$$\sum_{r=1}^{k+1}(2r-3) = k(k-2)+2(k+1)-3$$

(iv)
$$\begin{aligned}k(k-2)+2(k+1)-3 &= k^2-2k+2k+2-3\\ &= k^2-1\\ &= (k+1)(k-1) = (k+1)((k+1)-2)\end{aligned}$$

(v) So if the result is true for $n = k$, then it is true for $n = k + 1$.
Since it is true for $n = 1$, then it is true for all positive integers greater than or equal to 1 by induction.