Section 3: Matrices and simultaneous equations

Solutions to Exercise level 1

1. (i) $\det M = -2 - 3 = -5$

$$M^{-1} = -\frac{1}{5}\begin{pmatrix} -1 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 8 \\ -1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 8 \\ -1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

The solution is $x = 1, y = 2$

2. No unique solution if $\det\begin{pmatrix} a & 8 \\ 2 & a \end{pmatrix} = 0$

$$a^2 - 16 = 0$$

$$a = \pm 4$$

3. (i) $A^{-1} = \begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}$

(ii) $$\begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

The solution is $x = -1, y = 2, z = 1$.

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4. (i) $3x + 6y = a$

$-2x - 4y = b$

(ii) $\det\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix} = -12 + 12 = 0$

There is no unique solution to the equations in part (i).

If $2a = 3b$ the equations are consistent and have an infinite set of solutions.

If $2a \neq 3b$ then the equations are inconsistent and have no solution.