

Section 3: Invariance

Solutions to Exercise level 1

$$1. \quad (i) \quad \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2+3 \\ -3+6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$(ii) \quad \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} k \\ 3k \end{pmatrix} = \begin{pmatrix} -2k+3k \\ -3k+6k \end{pmatrix} = \begin{pmatrix} k \\ 3k \end{pmatrix}$$

$$(iii) \quad y = 3x$$

$$2. \quad (i) \quad \begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 4x+3y \\ -3x-2y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 3x+3y \\ -3x-3y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The invariant points are the points of the form $(\lambda, -\lambda)$.

$$(ii) \quad \begin{pmatrix} 0.5 & -0.5 \\ 0.5 & 1.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 0.5x-0.5y \\ 0.5x+1.5y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -0.5x-0.5y \\ 0.5x+0.5y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The invariant points are the points of the form $(\lambda, -\lambda)$.

$$3. \quad (i) \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.6x+0.8y \\ 0.8x-0.6y \end{pmatrix}$$

$$\text{For invariant points, } \begin{pmatrix} 0.6x+0.8y \\ 0.8x-0.6y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -0.4x+0.8y \\ 0.8x-1.6y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

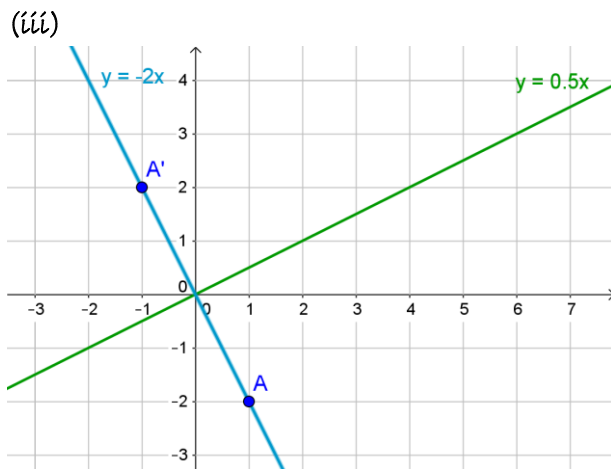
A point which is invariant is $(2, 1)$, or any point of the form $(2\lambda, \lambda)$.

The mirror line of the transformation is therefore the line $2y = x$.

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$$(ii) \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0.6 - 1.6 \\ 0.8 + 1.2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

so A' is the point $(-1, 2)$



(iv) Any lines of the form $y = -2x + c$ are invariant lines.