Section 2: The inverse of a 3×3 matrix

Solutions to Exercise level 3

1. $|P| = \begin{vmatrix} a & 1 & 1 \\ 0 & b & 1 \\ -3 & 0 & -1 \end{vmatrix} = a(-b-0)+1(-3-0)+1(0+3b)$

$= -ab-3+3b$

$|Q| = \begin{vmatrix} a+1 & 1 & 1 \\ 0 & b+1 & 1 \\ -3 & 0 & 0 \end{vmatrix} = (a+1)(0-0)+1(-3-0)+1(0+3(b+1))$

$= -3+3b+3$

$= 3b$

$|R| = \begin{vmatrix} a+2 & 1 & 1 \\ 0 & b+2 & 1 \\ -3 & 0 & 1 \end{vmatrix} = (a+2)(b+2-0)+1(-3-0)+1(0+3(b+2))$

$= ab+2b+2a+4-3+3b+6$

$= ab+5b+2a+7$

The scale factor of a transformation is given by the determinant of the matrix, so $|P| = |Q| = |R|$

$|P| = |Q| \Rightarrow -ab-3+3b = 3b$

$\Rightarrow ab = -3$

$|Q| = |R| \Rightarrow 3b = ab+5b+2a+7$

$\Rightarrow ab+2b+2a+7 = 0$

$\Rightarrow -3+2b+2a+7 = 0$

$\Rightarrow 2b+2a+4 = 0$

$\Rightarrow b = -a-2$

Substituting into $ab = -3$ gives

$(-a-2)a = -3$

$-a^2-2a = -3$

$a^2+2a-3 = 0$

$(a-1)(a+3) = 0$

$a = 1, b = -3$ or $a = -3, b = 1$

2. (i) There are six possibilities, since the non-zero elements must all be in different columns for the matrix to be non-singular.

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(ii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$ $\quad$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$ $\quad$ $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & \frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \\ 1 & 0 & 0 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 2 & 1 & \frac{2}{3} \\ 2 & 1 & \frac{2}{3} \\ 2 & 1 & \frac{2}{3} \end{pmatrix}$

Since all the rows are the same, the determinant is zero.