Section 1: Determinants and inverses

Solutions to Exercise level 2

1. $AX = B \Rightarrow X = A^{-1}B$

$\det A = \det\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = 1$

$A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

$X = A^{-1}B = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ 11 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix}$

$BY = C \Rightarrow Y = B^{-1}C$

$\det B = \det\begin{pmatrix} 6 & 1 \\ 11 & 3 \end{pmatrix} = 7$

$B^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}$

$Y = B^{-1}C = \frac{1}{7}\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}\begin{pmatrix} -4 & 3 \\ -5 & 2 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} -7 & 7 \\ 14 & -21 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$

$CZ = D \Rightarrow Z = C^{-1}D$

$\det C = \det\begin{pmatrix} -4 & 3 \\ -5 & 2 \end{pmatrix} = 7$

$C^{-1} = \frac{1}{7}\begin{pmatrix} 2 & -3 \\ 5 & -4 \end{pmatrix}$

$Z = C^{-1}D = \frac{1}{7}\begin{pmatrix} 2 & -3 \\ 5 & -4 \end{pmatrix}\begin{pmatrix} 4 & 7 \\ -2 & 7 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 14 & -7 \\ 28 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix}$

2. $\det\begin{pmatrix} a-3 & -2 \\ a & 2a-1 \end{pmatrix} = 0$

$(a-3)(2a-1) - (-2a) = 0$

$2a^2 - 7a + 3 + 2a = 0$

$2a^2 - 5a + 3 = 0$

$(2a-3)(a-1) = 0$

$a = \frac{3}{2}$ or 1

3. (i) $\det M = (2\times 1) - (2\times -3) = 2 + 6 = 8$

The area scale factor of the transformation is 8 so the area of T' is $4 \times 8 = 32$ square units

(ii) $M^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -2 \\ 3 & 2 \end{pmatrix}$

4. $\det M = (3\times 2) - (6 \times 1) = 6 - 6 = 0$

$$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+6y \\ x+2y \end{pmatrix} = \begin{pmatrix} 3(x+2y) \\ x+2y \end{pmatrix}$$

Every image point satisfies $x = 3y$, so all points are mapped to the line $x = 3y$.

5. (i) S is a two way stretch, with scale factor 2 in the x direction and scale factor 3 in the y direction,
R is a reflection in the y-axis.

(ii) $\det M = 6$

$$M^{-1} = \frac{1}{6}\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$\det N = -1$

$$N^{-1} = -1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(iii) $MN = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$

$\det MN = -6$

$$(MN)^{-1} = -\frac{1}{6}\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$N^{-1}M^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times \frac{1}{6}\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$
$$= \frac{1}{6}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$
$$= \frac{1}{6}\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix} = (MN)^{-1}$$

(iv) MN represents the transformation R (matrix N) followed by transformation S (matrix M). So to reverse this you need to first reverse S, using M^{-1}, and then reverse R using N^{-1}. So $N^{-1}M^{-1}$ is the inverse of MN.

6. $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+6y \\ -2x-4y \end{pmatrix} = \begin{pmatrix} 3(x+2y) \\ -2(x+2y) \end{pmatrix}$

$x' = 3(x+2y)$

$y' = -2(x+2y)$

$\frac{x'}{3} = \frac{y'}{-2}$

$-2x' = 3y'$

The matrix maps all points onto the straight line $2x+3y=0$.

7. (i) $\det\begin{pmatrix} 2 & -3 \\ 4 & -4 \end{pmatrix} = -8+12 = 4$

$A^{-1} = \frac{1}{4}\begin{pmatrix} -4 & 3 \\ -4 & 2 \end{pmatrix}$

(ii) $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 16 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 9 \\ 16 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -4 & 3 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 9 \\ 16 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 12 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

The point (3, -1) is mapped to (9, 16).

(iii) $A^2 = \begin{pmatrix} 2 & -3 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} -8 & 6 \\ -8 & 4 \end{pmatrix}$.

(iv) $A^3 = \begin{pmatrix} -8 & 6 \\ -8 & 4 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ so $d = 8$

(v) The matrix A^3 is an enlargement, centre the origin, scale factor 8.

8. (i) $\det A = 8-6 = 2$

$A^{-1} = \frac{1}{2}\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 2 & \frac{3}{2} \\ 1 & 1 \end{pmatrix}$

(ii) $A = BC$

$AC^{-1} = B$

$C^{-1} = A^{-1}B = \frac{1}{2}\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 10 & 17 \\ 6 & 10 \end{pmatrix} = \begin{pmatrix} 5 & 8.5 \\ 3 & 5 \end{pmatrix}$

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9. $A^2 = A^{-1} \Rightarrow A^3 = I$

$$A^2 = \begin{pmatrix} 1 & x & 1 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & x & 1 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2-x & 0 & 1 \\ 0 & 1-x & -1 \\ 1 & x & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2-x & 0 & 1 \\ 0 & 1-x & -1 \\ 1 & x & 1 \end{pmatrix}\begin{pmatrix} 1 & x & 1 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3-x & 2x-x^2 & 2-x \\ -2+x & -1+x & 0 \\ 2-x & 0 & 1 \end{pmatrix}$$

When $x = 2$, $A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

so the value of x is 2.