Section 1: Determinants and inverses

Solutions to Exercise level 1

1. (i) $\det A = (3\times 1)-(-1\times 2) = 3+2 = 5$

$$A^{-1} = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}$$

(ii) $\det B = (4\times 1)-(-2\times -2) = 4-4 = 0$

so the inverse of matrix B does not exist.

2. (i) $\begin{pmatrix} 0.125 & -0.1 & 0.175 \\ 0.125 & 0.3 & -0.025 \\ -0.25 & 0.4 & 0.05 \end{pmatrix}$ (ii) $\begin{pmatrix} 0.8 & -0.4 & -1 \\ -0.4 & 0.2 & 1 \\ 0.6 & 0.2 & -1 \end{pmatrix}$

3. $AB = I \Rightarrow A = B^{-1}$

$$\det B = \det\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} = 6-4 = 2$$

$$A = B^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}.$$

4. $AB = \begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix} \Rightarrow B = A^{-1}\begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix}$

$$\det A = \det\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = 6-2 = 4$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$$

$$B = A^{-1}\begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 12 & 8 \\ 4 & 12 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 3 \end{pmatrix}$$

5. $Y\begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow Y = \begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix}^{-1}$

$$\det\begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix} = -2-0 = -2$$

$$Y = \begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix}^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & -2 \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ 1.5 & 1 \end{pmatrix}$$

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6. $\begin{pmatrix} 1 & -2 \\ x & 4 \end{pmatrix}$ is singular $\Rightarrow \det\begin{pmatrix} 1 & -2 \\ x & 4 \end{pmatrix} = 0$

$\Rightarrow 4 + 2x = 0$

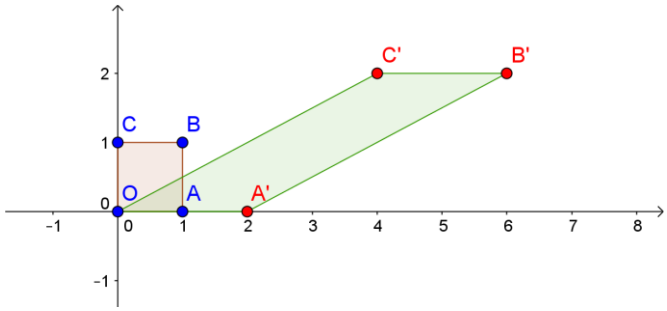
$\Rightarrow x = -2$

7. $PQ = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 6 & 5 \\ 5 & 7 & 6 \end{pmatrix}\begin{pmatrix} 1 & -4 & 3 \\ 1 & 2 & -2 \\ -2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$QP = \begin{pmatrix} 1 & -4 & 3 \\ 1 & 2 & -2 \\ -2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 & 2 \\ 4 & 6 & 5 \\ 5 & 7 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q is the inverse of P.

8. (i) $\begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 6 & 4 \\ 0 & 0 & 2 & 2 \end{pmatrix}$



(ii) $\det M = (2 \times 2) - (4 \times 0) = 4 - 0 = 4$

The area scale factor is 4, so the area of the image is 4 times the area of the original square. (Since the area of the original square is 1 square unit, the area of the image is 4 square units)