Section 1: Introduction to matrices

Solutions to Exercise level 2

1. In the second half of the season:
A win two and lose two
B win one and lose three
C win two and draw two
D win one, draw one and lose two
E win two, draw one and lose one

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \end{array}$$

The results matrix for the whole season is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 4 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} \end{array} + \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 4 & 1 & 3 \\ 3 & 1 & 4 \\ 2 & 6 & 0 \\ 2 & 2 & 4 \\ 3 & 2 & 3 \end{pmatrix} \end{array}$$

A's points are $(4\times3)+(1\times1)=13$
B's points are $(3\times3)+(1\times1)=10$
C's points are $(2\times3)+(6\times1)=12$
D's points are $(2\times3)+(2\times1)=8$
E's points are $(3\times3)+(2\times1)=11$

The league positions are A, C, E, B, D.

2. (i) $\begin{pmatrix} 3 & -5 \\ 2 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & -2 \end{pmatrix}$

$\begin{pmatrix} 4 & -5+y \\ 5 & x+2 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & -2 \end{pmatrix}$

$-5+y=6 \quad \Rightarrow y=11$

$x+2=-2 \quad \Rightarrow x=-4$

(ii) $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + x\begin{pmatrix} -2 \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \end{pmatrix}$

$\begin{pmatrix} 3-2x \\ -1+xy \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \end{pmatrix}$

$3-2x=-5 \quad \Rightarrow x=4$

$-1+xy=11 \quad \Rightarrow -1+4y=11 \quad \Rightarrow y=3$

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3. (a) (i)

$$(A+B)^2 = \left[\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}\right]^2$$

$$= \begin{pmatrix} -1 & 1 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 5 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -2 \\ -10 & 6 \end{pmatrix}$$

(ii)

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -7 & 2 \end{pmatrix}$$

(iii)

$$BA = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$$

(b)

$$A^2 = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ -6 & 3 \end{pmatrix}$$

$$A^2 + AB + BA + B^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ -7 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 7 & -2 \\ -6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -2 \\ -10 & 6 \end{pmatrix}$$

$$= (A+B)^2$$

4. $A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 4 & 4 \end{pmatrix}$$

$$A^2 = hA + kI \Rightarrow \begin{pmatrix} 4 & 0 \\ 4 & 4 \end{pmatrix} = h\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4 & 0 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 2h+k & 0 \\ h & 2h+k \end{pmatrix}$$

$$\Rightarrow h = 4, k = -4$$

5. Total value of sales for each garage is given by

$$\begin{matrix} G_1 \\ G_2 \\ G_3 \end{matrix}\begin{pmatrix} 3 & 1 \\ 2 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 8000 \\ 10500 \end{pmatrix} = \begin{matrix} G_1 \\ G_2 \\ G_3 \end{matrix}\begin{pmatrix} 3\times 8000 + 1\times 10500 \\ 2\times 8000 + 0\times 10500 \\ 4\times 8000 + 1\times 10500 \end{pmatrix} = \begin{matrix} G_1 \\ G_2 \\ G_3 \end{matrix}\begin{pmatrix} 34500 \\ 16000 \\ 42500 \end{pmatrix}$$

