Section 1: Introduction to matrices

Solutions to Exercise level 1

1. (i)
$$\begin{matrix} & A & B & C \\ A & 0 & 1 & 0 \\ B & 1 & 0 & 2 \\ C & 0 & 2 & 0 \end{matrix}$$

(ii)
$$\begin{matrix} & A & B & C \\ A & 0 & 1 & 2 \\ B & 1 & 0 & 1 \\ C & 2 & 1 & 0 \end{matrix}$$

2. Multiply the A row by 4, the B row by 5 and the C row by 3.

$$\begin{matrix} & M & T \\ A & 4\times4 & 4\times6 \\ B & 5\times3 & 5\times5 \\ C & 3\times7 & 3\times2 \end{matrix} = \begin{matrix} & M & T \\ A & 16 & 24 \\ B & 15 & 25 \\ C & 21 & 6 \end{matrix}$$

The total taken for meal A over the two days is £16 + £24 = £40.
The total taken for meal B over the two days is £15 + £25 = £40.
The total taken for meal C over the two days is £21 + £6 = £27.

3. (i) $A+2B=\begin{pmatrix}2&-3\\-1&5\end{pmatrix}+2\begin{pmatrix}-3&-1\\2&7\end{pmatrix}=\begin{pmatrix}2&-3\\-1&5\end{pmatrix}+\begin{pmatrix}-6&-2\\4&14\end{pmatrix}$
$=\begin{pmatrix}-4&-5\\3&19\end{pmatrix}$

(ii) $C-D=\begin{pmatrix}2&3&-4\\-1&2&5\end{pmatrix}-\begin{pmatrix}-1&-4&2\\-3&5&6\end{pmatrix}$
$=\begin{pmatrix}3&7&-6\\2&-3&-1\end{pmatrix}$

(iii) cannot be done as A and C do not have the same order

(iv) $3D-C=3\begin{pmatrix}-1&-4&2\\-3&5&6\end{pmatrix}-\begin{pmatrix}2&3&-4\\-1&2&5\end{pmatrix}$
$=\begin{pmatrix}-3&-12&6\\-9&15&18\end{pmatrix}-\begin{pmatrix}2&3&-4\\-1&2&5\end{pmatrix}$
$=\begin{pmatrix}-5&-15&10\\-8&13&13\end{pmatrix}$

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4. (i) $\mathbf{AB} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 2 \\ 23 & -5 & -14 \end{pmatrix}$

(ii) $\mathbf{AC} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ -5 & 11 \end{pmatrix}$

(iii) BC cannot be calculated as the matrices are not conformable (the number of columns in B is not the same as the number of rows in C)

(iv) $\mathbf{BD} = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 18 \\ 28 & -2 \end{pmatrix}$

5. (i) $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 6 & 2 \end{pmatrix}$

$\mathbf{AB} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 6 \\ 6 & 9 \end{pmatrix}$

(ii) $\mathbf{A} + \mathbf{B} - \mathbf{AB} = \begin{pmatrix} 2 & 6 \\ 6 & 2 \end{pmatrix} - \begin{pmatrix} 9 & 6 \\ 6 & 9 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = -7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -7\mathbf{I}$

6. (i) $2\mathbf{A} + \mathbf{C} = 2\begin{pmatrix} 1 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 8 \end{pmatrix} + \begin{pmatrix} 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 4 & 7 \end{pmatrix}$

(ii) $\mathbf{AB} = \begin{pmatrix} 1 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \end{pmatrix}$

(iii) $\mathbf{BC} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 4 & -2 \end{pmatrix}$

7. $\mathbf{AB} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}\begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix} = \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix}$

$\mathbf{BA} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}\begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$

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$$AB = BA \Rightarrow \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$$

$22 = 18 + 2x \quad \Rightarrow x = 2$

$6 + y = 10 \quad \Rightarrow y = 4$

Check: $6x + 8 = 12 + 8 = 20$ $\quad 12 + xy = 12 + 8 = 20$

$2x + 2y = 4 + 8 = 12$ $\quad 4 + 2y = 4 + 8 = 12$

8. $M^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$N^2 = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 6 & -10 \\ -5 & 11 \end{pmatrix}$

$M^2 - N^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6 & -10 \\ -5 & 11 \end{pmatrix} = \begin{pmatrix} -5 & 14 \\ 5 & -10 \end{pmatrix}$

$M + N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}$

$M - N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$

$(M + N)(M - N) = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 12 \\ 5 & -12 \end{pmatrix}$

$(M + N)(M - N) = M^2 + NM - MN - N^2$

Since matrix multiplication is not commutative, $NM \neq MN$.