

## Section 1: Volumes of revolution

### Solutions to Exercise level 1

$$\begin{aligned}
 1. \quad (i) \quad \text{volume} &= \pi \int_0^1 y^2 dx \\
 &= \pi \int_0^1 (x^3)^2 dx \\
 &= \pi \int_0^1 x^6 dx \\
 &= \pi \left[ \frac{1}{7} x^7 \right]_0^1 \\
 &= \pi \left( \frac{1}{7} - 0 \right) \\
 &= \frac{1}{7} \pi
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \text{volume} &= \pi \int_0^4 (\sqrt{x})^2 dx \\
 &= \pi \int_0^4 x dx \\
 &= \pi \left[ \frac{1}{2} x^2 \right]_0^4 \\
 &= \pi \left( \frac{1}{2} \times 16 - 0 \right) \\
 &= 8\pi
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \text{volume} &= \pi \int_{-1}^1 y^2 dx \\
 &= \pi \int_{-1}^1 (1 - x^2)^2 dx \\
 &= \pi \int_{-1}^1 (1 - 2x^2 + x^4) dx \\
 &= \pi \left[ x - \frac{2}{3} x^3 + \frac{1}{5} x^5 \right]_{-1}^1 \\
 &= \pi \left( 1 - \frac{2}{3} + \frac{1}{5} - \left( -1 + \frac{2}{3} - \frac{1}{5} \right) \right) \\
 &= \frac{16}{15} \pi
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad \text{volume} &= \pi \int_1^2 \left( 1 - \frac{1}{x^2} \right)^2 dx \\
 &= \pi \int_1^2 (1 - 2x^{-2} + x^{-4}) dx \\
 &= \pi \left[ x + 2x^{-1} - \frac{1}{3} x^{-3} \right]_1^2 \\
 &= \pi \left[ x + \frac{2}{x} - \frac{1}{3x^3} \right]_1^2 \\
 &= \pi \left( 2 + 1 - \frac{1}{24} - \left( 1 + 2 - \frac{1}{3} \right) \right) \\
 &= \frac{7}{24} \pi
 \end{aligned}$$

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$$\begin{aligned} \text{(v) volume} &= \pi \int_{-1}^2 (x^2 + 1)^2 dx \\ &= \pi \int_{-1}^2 (x^4 + 2x^2 + 1)^2 dx \\ &= \pi \left[ \frac{1}{5} x^5 + \frac{2}{3} x^3 + x \right]_{-1}^2 \\ &= \pi \left( \frac{32}{5} + \frac{16}{3} + 2 - \left( -\frac{1}{5} - \frac{2}{3} - 1 \right) \right) \\ &= \frac{78}{5} \pi \end{aligned}$$

2. (i)  $y = x^3 \Rightarrow x = y^{\frac{1}{3}}$

$$\begin{aligned} \text{volume} &= \pi \int_0^1 x^2 dy \\ &= \pi \int_0^1 (y^{\frac{1}{3}})^2 dy \\ &= \pi \int_0^1 y^{\frac{2}{3}} dy \\ &= \pi \left[ \frac{3}{5} y^{\frac{5}{3}} \right]_0^1 \\ &= \pi \left( \frac{3}{5} - 0 \right) \\ &= \frac{3}{5} \pi \end{aligned}$$

(ii)  $y = \sqrt{x} \Rightarrow x = y^2$

$$\begin{aligned} \text{volume} &= \pi \int_0^2 (y^2)^2 dy \\ &= \pi \int_0^2 y^4 dy \\ &= \pi \left[ \frac{1}{5} y^5 \right]_0^2 \\ &= \pi \left( \frac{32}{5} - 0 \right) \\ &= \frac{32}{5} \pi \end{aligned}$$

(iii)  $y = 1 - x^2 \Rightarrow x^2 = 1 - y$

$$\begin{aligned} \text{volume} &= \pi \int_0^1 x^2 dy \\ &= \pi \int_0^1 (1 - y) dy \\ &= \pi \left[ y - \frac{1}{2} y^2 \right]_0^1 \\ &= \pi \left( 1 - \frac{1}{2} - 0 \right) \\ &= \frac{1}{2} \pi \end{aligned}$$

$$\text{(iv) } y = \frac{1}{\sqrt{1+x^2}} \Rightarrow \sqrt{1+x^2} = \frac{1}{y} \Rightarrow 1+x^2 = \frac{1}{y^2} \Rightarrow x^2 = \frac{1}{y^2} - 1$$

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$$\begin{aligned}\text{volume} &= \pi \int_{\frac{1}{2}}^1 x^2 dy \\ &= \pi \int_{\frac{1}{2}}^1 \left( \frac{1}{y^2} - 1 \right) dy \\ &= \pi \left[ -\frac{1}{y} - y \right]_{\frac{1}{2}}^1 \\ &= \pi \left( -1 - 1 - \left( -2 - \frac{1}{2} \right) \right) \\ &= \frac{1}{2} \pi\end{aligned}$$

$$(v) \quad y = \sqrt{x+1} \Rightarrow y^2 = x+1 \Rightarrow x = y^2 - 1$$

$$\begin{aligned}\text{volume} &= \pi \int_1^2 x^2 dy \\ &= \pi \int_1^2 (y^2 - 1)^2 dy \\ &= \pi \int_1^2 (y^4 - 2y^2 + 1) dy \\ &= \pi \left[ \frac{1}{5} y^5 - \frac{2}{3} y^3 + y \right]_1^2 \\ &= \pi \left( \frac{32}{5} - \frac{16}{3} + 2 - \left( \frac{1}{5} - \frac{2}{3} + 1 \right) \right) \\ &= \frac{38}{15} \pi\end{aligned}$$