Section 2: Complex roots of polynomials

Solutions to Exercise level 1

1. (i) The other root is $4+5i$

   (ii) $z = 4 \pm 5i$

   $z - 4 = 5i$

   $(z-4)^2 = (5i)^2$

   $z^2 - 8z + 16 = -25$

   $z^2 - 8z + 41 = 0$

   so $p = -8, q = 41$

2. (i) Substituting $z = 1+i$:

   $z^3 - 2z + 4 = (1+i)^3 - 2(1+i) + 4$

   $= 1 + 3i + 3i^2 + i^3 - 2 - 2i + 4$

   $= 1 + 3i - 3 - i - 2 - 2i + 4$

   $= 1 - 3 - 2 + 4 + (3 - 1 - 2)i$

   $= 0$

   so $z = 1+i$ is a root

   (ii) The other complex root is $z = 1-i$

   (iii) The sum of the roots is 0 since the coefficient of z^2 is zero.

   Let the third root be α

   $1 + i + 1 - i + \alpha = 0$

   $2 + \alpha = 0$

   $\alpha = -2$

   so the other root is $z = -2$

3. 1 – 2i is a root, so 1 + 2i is also a root.

   The sum of the roots is 0 since the coefficient of z^2 is zero.

   Let the third root be α

   $1 + 2i + 1 - 2i + \alpha = 0$

   $2 + \alpha = 0$

   $\alpha = -2$

   The real root is $z = -2$.

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4. Since $1+3i$ is a root, $1-3i$ is another root.
   The sum of the roots is 3
   Let the third root be α

   $1+3i+1-3i+\alpha=3$

   $2+\alpha=3$

   $\alpha=1$

   So the third root is $z=1$

5. $z=2$ is a root so $(z-2)$ is a factor

   $z^3-4z^2+6z-4=0$

   $(z-2)(z^2-2z+2)=0$

   $$z^2-2z+2=0 \Rightarrow z=\frac{-(-2)\pm\sqrt{(-2)^2-4\times1\times2}}{2\times1}$$

   $$\Rightarrow z=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm2i}{2}=1\pm i$$