Section 1: Roots and coefficients

Solutions to Exercise level 1

1. (i) $2x^2 + 9x - 5 = 0$

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{9}{2}$$

$$\text{Product of roots} = \frac{c}{a} = \frac{-5}{2} = -\frac{5}{2}$$

(ii) $5x^2 - x + 2 = 0$

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{-1}{5} = \frac{1}{5}$$

$$\text{Product of roots} = \frac{c}{a} = \frac{2}{5}$$

(iii) $3x(x+2) = 4x - 5$

$$3x^2 + 6x = 4x - 5$$

$$3x^2 + 2x + 5 = 0$$

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{2}{3}$$

$$\text{Product of roots} = \frac{c}{a} = \frac{5}{3}$$

2. (i) $x^3 - 3x^2 + 2x + 4 = 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{2}{1} = 2$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{4}{1} = -4$$

(ii) $2x^3 + 5x - 3 = 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{0}{2} = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{5}{2}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-3}{2} = \frac{3}{2}$$

(iii) $3x^3 + x^2 - 4x - 1 = 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{1}{3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-4}{3} = -\frac{4}{3}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-1}{3} = \frac{1}{3}$$

3. $3x^2 + 11x - 4 = 0$

$$\alpha + \beta = -\frac{11}{3}$$

$$\alpha\beta = -\frac{4}{3}$$

(i) For new equation,

sum of roots $= \alpha - 2 + \beta - 2 = (\alpha + \beta) - 4$

$= -\frac{11}{3} - 4 = -\frac{23}{3}$

and product of roots $= (\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$

$= -\frac{4}{3} + \frac{22}{3} + 4 = 10$

So for new equation, $-\frac{b}{a} = -\frac{23}{3}$ and $\frac{c}{a} = 10$

Taking $a = 3$ gives $b = 23$ and $c = 30$

The new equation is $3x^2 + 23x + 30 = 0$

(ii) For new equation,

sum of roots $= 3\alpha + 3\beta = 3(\alpha + \beta)$

$= 3 \times -\frac{11}{3} = -11$

and product of roots $= 3\alpha \times 3\beta = 9\alpha\beta$

$= 9 \times -\frac{4}{3} = -12$

So for new equation, $-\frac{b}{a} = -11$ and $\frac{c}{a} = -12$

Taking $a = 1$ gives $b = 11$ and $c = -12$

The new equation is $x^2 + 11x - 12 = 0$

4. $p + q = 5, \quad p^2 + q^2 = 19$

$(p + q)^2 = p^2 + q^2 + 2pq$

$5^2 = 19 + 2pq$

$6 = 2pq$

$pq = 3$

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Sum of roots $= -\frac{b}{a} \Rightarrow 5 = -\frac{b}{a} \Rightarrow b = -5a$

Product of roots $= \frac{c}{a} \Rightarrow 3 = \frac{c}{a} \Rightarrow c = 3a$

Putting $a = 1$ gives $b = -5$ and $c = 3$

A quadratic equation with roots p and q is $x^2 - 5x + 3 = 0$

5. $x^2 + x - 6 = 0$

$\alpha + \beta = -1$

$\alpha\beta = -6$

$$\alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = \alpha + \beta + \frac{\beta + \alpha}{\alpha\beta}$$

$$= -1 + \frac{-1}{-6}$$

$$= -1 + \tfrac{1}{6}$$

$$= -\tfrac{5}{6}$$

6. Sum of roots = -5

$-1 + 4 + \alpha = -5$

$\alpha = -8$

The third root is -8.

$\sum \alpha\beta = a$

$(-1 \times 4) + (4 \times -8) + (-8 \times -1) = a$

$-4 - 32 + 8 = a$

$a = -28$

$\alpha\beta\gamma = -b$

$-1 \times 4 \times -8 = -b$

$b = -32$