

Section 1: Summing series

Solutions to Exercise level 3

$$\begin{aligned}
 1. \quad 2^2 + 5^2 + 8^2 + 11^2 + \dots &= \sum_{r=1}^n (3r-1)^2 = \sum_{r=1}^n (9r^2 - 6r + 1) \\
 &= 9 \sum_{r=1}^n r^2 - 6 \sum_{r=1}^n r + \sum_{r=1}^n 1 \\
 &= 9 \times \frac{1}{6} n(n+1)(2n+1) - 6 \times \frac{1}{2} n(n+1) + n \\
 &= \frac{3}{2} n(n+1)(2n+1) - 3n(n+1) + n \\
 &= \frac{1}{2} n [3(n+1)(2n+1) - 6(n+1) + 2] \\
 &= \frac{1}{2} n (6n^2 + 9n + 3 - 6n - 6 + 2) \\
 &= \frac{1}{2} n (6n^2 + 3n - 1)
 \end{aligned}$$

$$\begin{aligned}
 2. \quad u_r &= S_r - S_{r-1} \\
 &= 2r^2 + 7r - (2(r-1)^2 + 7(r-1)) \\
 &= 2r^2 + 7r - (2r^2 - 4r + 2 + 7r - 7) \\
 &= 2r^2 + 7r - (2r^2 + 3r - 5) \\
 &= 2r^2 + 7r - 2r^2 - 3r + 5 \\
 &= 4r + 5
 \end{aligned}$$

$$\begin{aligned}
 \sum_{r=n}^{2n} u_r &= \sum_{r=1}^{2n} u_r - \sum_{r=1}^{n-1} u_r \\
 &= S_{2n} - S_{n-1} \\
 &= 2(2n)^2 + 7(2n) - (2(n-1)^2 + 7(n-1)) \\
 &= 8n^2 + 14n - (2n^2 - 4n + 2 + 7n - 7) \\
 &= 8n^2 + 14n - (2n^2 + 3n - 5) \\
 &= 6n^2 + 11n + 5 \\
 &= (6n + 5)(n + 1)
 \end{aligned}$$