Section 1: Summing series

Solutions to Exercise level 2

1. (i) $$\sum_{r=30}^{50}(r^3-2)=\sum_{r=1}^{50}(r^3-2)-\sum_{r=1}^{29}(r^3-2)$$

$$\sum_{r=1}^{50}(r^3-2)=\sum_{r=1}^{50}r^3-\sum_{r=1}^{50}2$$
$$=\left(\tfrac{1}{4}\times 50^2\times 51^2\right)-(50\times 2)$$
$$=1625625-100$$
$$=1625525$$

$$\sum_{r=1}^{29}(r^3-2)=\sum_{r=1}^{29}r^3-\sum_{r=1}^{29}2$$
$$=\left(\tfrac{1}{4}\times 29^2\times 30^2\right)-(29\times 2)$$
$$=189225-58$$
$$=189167$$

$$\sum_{r=30}^{50}(r^3-2)=1625525-189167=1436358$$

(ii) $$\sum_{r=10}^{20}r(r-2)=\sum_{r=1}^{20}r(r-2)-\sum_{r=1}^{9}r(r-2)$$

$$\sum_{r=1}^{20}r^2-2r=\sum_{r=1}^{20}r^2-2\sum_{r=1}^{20}r$$
$$=\left(\tfrac{1}{6}\times 20\times 21\times 41\right)-2\left(\tfrac{1}{2}\times 20\times 21\right)$$
$$=2870-420$$
$$=2450$$

$$\sum_{r=1}^{9}r^2-2r=\sum_{r=1}^{9}r^2-2\sum_{r=1}^{9}r$$
$$=\left(\tfrac{1}{6}\times 9\times 10\times 19\right)-2\left(\tfrac{1}{2}\times 9\times 10\right)$$
$$=285-90$$
$$=195$$

$$\sum_{r=10}^{20}r(r-2)=2450-195=2255$$

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2. $$\sum_{r=1}^{n}(r+1)(2r+1)=\sum_{r=1}^{n}(2r^2+3r+1)$$
$$=2\sum_{r=1}^{n}r^2+3\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$
$$=2\times\tfrac{1}{6}n(n+1)(2n+1)+3\times\tfrac{1}{2}n(n+1)+n$$
$$=\tfrac{1}{6}n\left(4n^2+6n+2+9n+9+6\right)$$
$$=\tfrac{1}{6}n(4n^2+15n+17)$$
Putting $n=20$:
$2\times3+3\times5+4\times7+\ldots\ldots\ldots+21\times41$
$$=\tfrac{1}{6}\times20(4\times20^2+15\times20+17)$$
$$=6390$$

3. $$\sum_{r=1}^{n}(r+4)^3=\sum_{r=1}^{n+4}r^3-\sum_{r=1}^{r=4}r^3$$
$$=\tfrac{1}{4}(n+4)^2(n+5)^2-\tfrac{1}{4}\times4^2\times5^2$$
$$=\tfrac{1}{4}\left((n^2+9n+20)^2-400\right)$$
$$=\tfrac{1}{4}\left(n^4+18n^3+121n^2+360n+400-400\right)$$
$$=\tfrac{1}{4}n(n^3+18n^2+121n+360)$$

4. The rth term is $2r(2r-1)$.
$$\sum_{r=1}^{n}(2r(2r-1))=4\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r$$
$$=4\times\tfrac{1}{6}n(n+1)(2n+1)-2\times\tfrac{1}{2}n(n+1)$$
$$=\tfrac{1}{3}n(n+1)(4n+2-3)$$
$$=\tfrac{1}{3}n(n+1)(4n-1)$$

5. $$\sum_{1}^{n}r^3=\tfrac{1}{4}n^2(n+1)^2$$
$$\sum_{r=n+1}^{2n}r^3=\sum_{r=1}^{2n}r^3-\sum_{r=1}^{n}r^3$$
$$=\tfrac{1}{4}(2n)^2(2n+1)^2-\tfrac{1}{4}n^2(n+1)^2$$
$$=\tfrac{1}{4}n^2\left(4(2n+1)^2-(n+1)^2\right)$$
$$=\tfrac{1}{4}n^2(16n^2+16n+4-n^2-2n-1)$$
$$=\tfrac{1}{4}n^2(15n^2+14n+3)$$
$$=\tfrac{1}{4}n^2(3n+1)(5n+3)$$