Section 1: Summing series

Solutions to Exercise level 1

1. (i) $\sum_{r=1}^{20} 5 = 20 \times 5 = 100$

 (ii) $\sum_{r=1}^{100} r = \frac{1}{2} \times 100 \times 101 = 5050$

 (iii) $\sum_{r=1}^{50} r^2 = \frac{1}{6} \times 50 \times 51 \times 101 = 42925$

 (iv) $\sum_{r=1}^{20} r^3 = \frac{1}{4} \times 20^2 \times 21^2 = 44100$

2. (i)
$$\begin{aligned}\sum_{r=1}^{20} (2r^2 - 1) &= 2\sum_{r=1}^{20} r^2 - \sum_{r=1}^{20} 1 \\ &= 2\left(\tfrac{1}{6} \times 20 \times 21 \times 41\right) - (20 \times 1) \\ &= 5740 - 20 \\ &= 5720\end{aligned}$$

 (ii)
$$\begin{aligned}\sum_{r=1}^{10} r(r-1)^2 &= \sum_{r=1}^{10} (r^3 - 2r^2 + r) \\ &= \sum_{r=1}^{10} r^3 - 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r \\ &= \left(\tfrac{1}{4} \times 10^2 \times 11^2\right) - 2\left(\tfrac{1}{6} \times 10 \times 11 \times 21\right) + \left(\tfrac{1}{2} \times 10 \times 11\right) \\ &= 3025 - 770 + 55 \\ &= 2310\end{aligned}$$

3. (i)
$$\begin{aligned}\sum_{r=1}^{n} (2r - 1) &= 2\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 \\ &= 2 \times \tfrac{1}{2} n(n+1) - n \\ &= n(n+1) - n \\ &= n^2 + n - n \\ &= n^2\end{aligned}$$

 (ii)
$$\begin{aligned}\sum_{r=1}^{n} r(3r + 1) &= 3\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r \\ &= 3 \times \tfrac{1}{6} n(n+1)(2n+1) + \tfrac{1}{2} n(n+1) \\ &= \tfrac{1}{2} n(n+1)(2n+1+1) \\ &= \tfrac{1}{2} n(n+1)(2n+2) \\ &= n(n+1)^2\end{aligned}$$

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(iii) $$\sum_{r=1}^{n}(r+1)r^2 = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$$
$$= \tfrac{1}{4}n^2(n+1)^2 + \tfrac{1}{6}n(n+1)(2n+1)$$
$$= \tfrac{1}{12}n(n+1)[3n(n+1)+2(2n+1)]$$
$$= \tfrac{1}{12}n(n+1)(3n^2+7n+2)$$
$$= \tfrac{1}{12}n(n+1)(n+2)(3n+1)$$

(iv) $$\sum_{r=1}^{n}(4r^3-6r^2+4r-1) = 4\sum_{r=1}^{n} r^3 - 6\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$
$$= n^2(n+1)^2 - n(n+1)(2n+1) + 2n(n+1) - n$$
$$= n\left[n(n+1)^2 - (n+1)(2n+1) + 2(n+1) - 1\right]$$
$$= n\left[n^3+2n^2+n-2n^2-3n-1+2n+2-1\right]$$
$$= n(n^3)$$
$$= n^4$$

4. $$\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$
$$= \tfrac{1}{6}n(n+1)(2n+1) + \tfrac{1}{2}n(n+1)$$
$$= \tfrac{1}{6}n(n+1)[2n+1+3]$$
$$= \tfrac{1}{6}n(n+1)(2n+4)$$
$$= \tfrac{1}{3}n(n+1)(n+2)$$

5. $$\sum_{r=1}^{n}(2r-1)^2 = \sum_{r=1}^{n}(4r^2-4r+1)$$
$$= 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$
$$= 4\times\tfrac{1}{6}n(n+1)(2n+1) - 4\times\tfrac{1}{2}n(n+1) + n$$
$$= \tfrac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$$
$$= \tfrac{1}{3}n[2(n+1)(2n+1) - 6(n+1) + 3]$$
$$= \tfrac{1}{3}n(4n^2+6n+2-6n-6+3)$$
$$= \tfrac{1}{3}n(4n^2-1)$$
$$= \tfrac{1}{3}n(2n+1)(2n-1)$$

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6. $(2\times3)+(3\times4)+(4\times5)+.......+(n+1)(n+2)=\sum_{r=1}^{n}(r+1)(r+2)$

$$\begin{aligned}\sum_{r=1}^{n}(r+1)(r+2) &= \sum_{r=1}^{n}(r^2+3r+2)\\
&= \sum_{r=1}^{n}r^2+3\sum_{r=1}^{n}r+\sum_{r=1}^{n}2\\
&= \tfrac{1}{6}n(n+1)(2n+1)+3\times\tfrac{1}{2}n(n+1)+2n\\
&= \tfrac{1}{6}n[(n+1)(2n+1)+9(n+1)+12]\\
&= \tfrac{1}{6}n[2n^2+3n+1+9n+9+12]\\
&= \tfrac{1}{6}n(2n^2+12n+22)\\
&= \tfrac{1}{3}n(n^2+6n+11)\end{aligned}$$

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