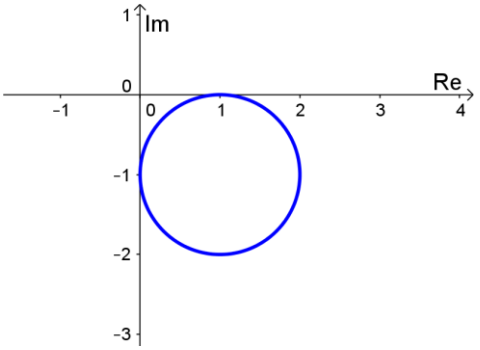
Section 2: Loci in the complex plane

Solutions to Exercise level 2

1. (i) $|z-1+i|=1$

$|z-(1-i)|=1$

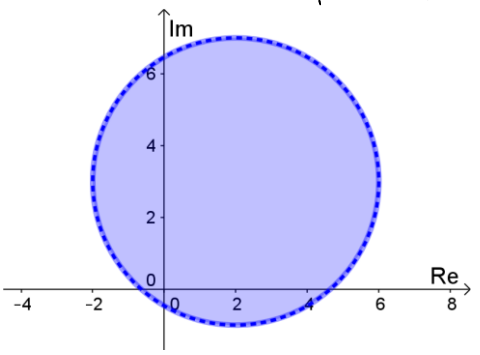
This is a circle, centre 1 – i, radius 1.



(ii) $|z-2-3i|<4$

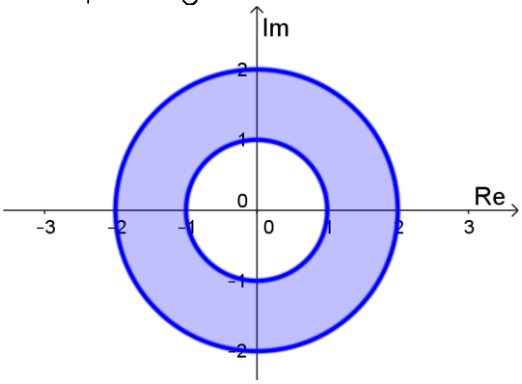
$|z-(2+3i)|<4$

This is the interior of a circle, centre 2 + 3i, radius 4.



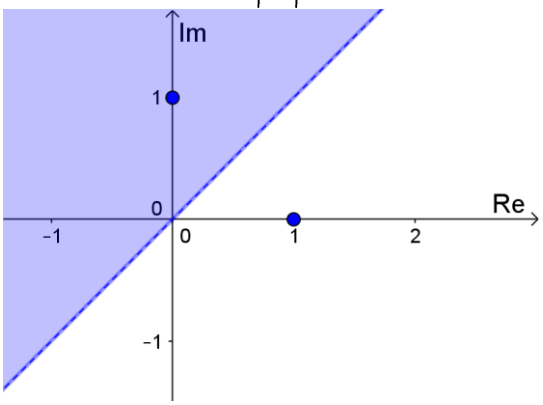
(iii) $1 \leq |z| \leq 2$

This is the region in between two circles, centres the origin, radii 1 and 2 respectively. The two circles are included.

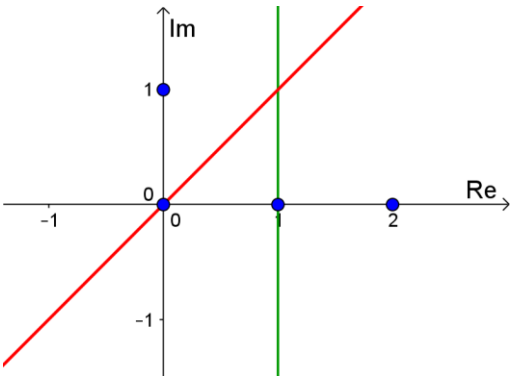


(iv) $|z-1|>|z-i|$

The boundary of this set of points is the perpendicular bisector of the points 1 and i. The locus is the region closer to the point i than the point 1, and does not include the perpendicular bisector itself.

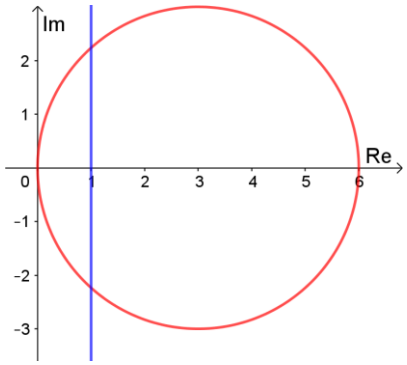


2.



The point of intersection of the lines $x=1$ and $y=x$ is (1, 1)
so the value of z that satisfies both equations is $1+i$.

3. $|z-3|=3$ is a circle, centre 3, radius 3.
$|z|=|z-2|$ is the perpendicular bisector of the line joining the origin and the point (2, 0).



The equation of the circle is $(x-3)^2+y^2=9$
The equation of the line is $x=1$.

At the points of intersection,

$(1-3)^2+y^2=9$

$4+y^2=9$

$y=\pm\sqrt{5}$

The complex numbers satisfying both equations are $1+\sqrt{5}i$ and $1-\sqrt{5}i$.

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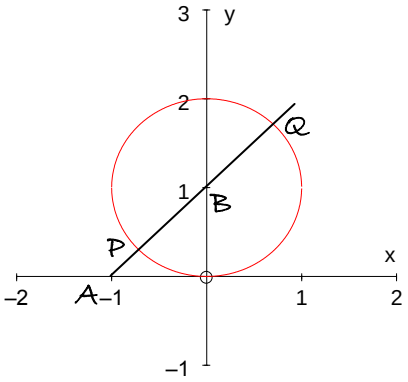
4. The point z lies on a circle, centre (0, 1) and radius 1.
The value of $|z + 1|$ is the distance of the point z from the point (-1, 0).

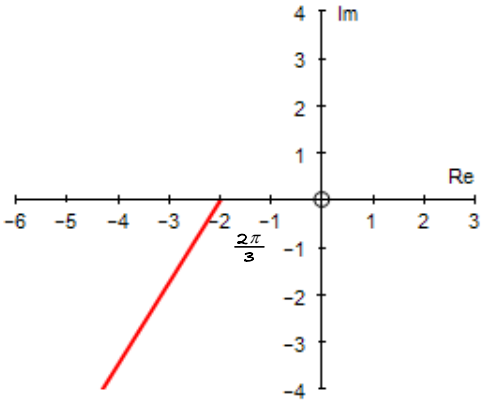
The diagram shows the points P and Q which give the least and greatest values of $|z + 1|$ respectively.



The distance $AB = \sqrt{1^2 + 1^2} = \sqrt{2}$
$PB = 1$ so $AP = \sqrt{2} - 1$.
$AQ = 1 + AB = \sqrt{2} + 1$.
The greatest and least values of $|z + 1|$ are $\sqrt{2} + 1$ and $\sqrt{2} - 1$.

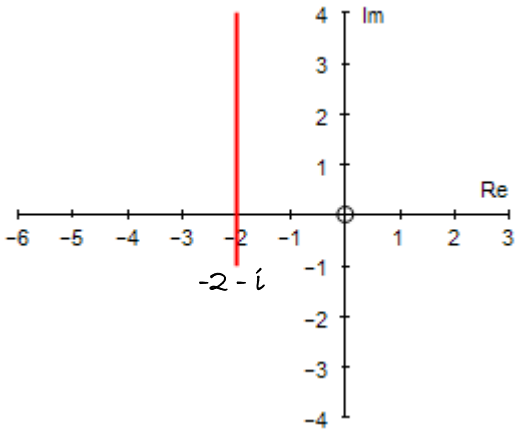
5. (i) $\arg(z + 2) = -\frac{2\pi}{3}$

This is a half-line, starting at -2, at an angle of $\frac{2\pi}{3}$ below the positive real axis.



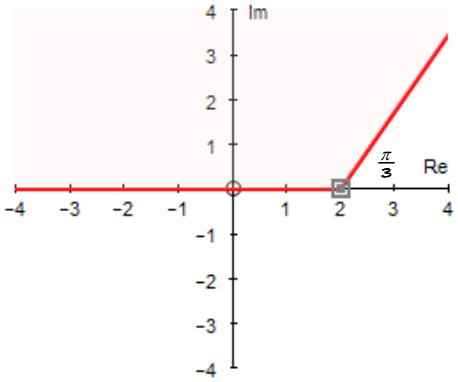
(ii) $\arg(z + 2 + i) = \frac{\pi}{2}$

This is a half-line, starting at -2 – i, at an angle of $\frac{\pi}{2}$ above the positive real axis.

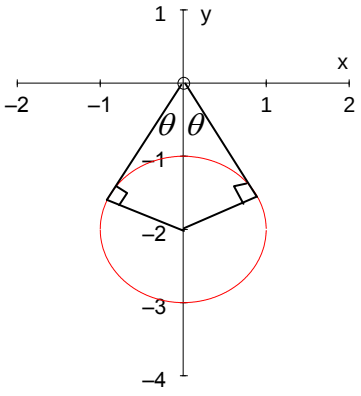


(iii) $\frac{\pi}{3}$

The boundaries of this region are two half-lines, both starting at 2, one at an angle to $\frac{\pi}{3}$ to the positive real axis, and the other on the negative real axis.



6. $|z+2i|=1$ is a circle, centre -2i and radius 1.



From diagram, $\sin\theta = \frac{1}{2}$

$\theta = 30°$

The greatest value of arg z is $-\frac{\pi}{3}$

The least value of arg z is $-\frac{2\pi}{3}$

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7. $|z+2+i|=|z-4+i|$

$(x+2)^2+(y+1)^2=(x-4)^2+(y+1)^2$

$x^2+4x+4=x^2-8x+16$

$12x=12$

$x=1$

$y=x\tan\theta=x\tan\frac{\pi}{4}=x$

When $x=1$, $y=1$

so the complex number is $1+i$.