Edexcel AS Further Maths Complex numbers

Section 1: Modulus and argument

Solutions to Exercise level 1

1. (i) $|z| = |4-3i| = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

   (ii) $2w = 2(1+2i) = 2+4i$

   $|2w| = |2+4i| = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

   (ii) $\dfrac{z}{w} = \dfrac{4-3i}{1+2i} = \dfrac{(4-3i)(1-2i)}{(1+2i)(1-2i)} = \dfrac{4-3i-8i-6}{1+4} = \dfrac{-2-11i}{5}$

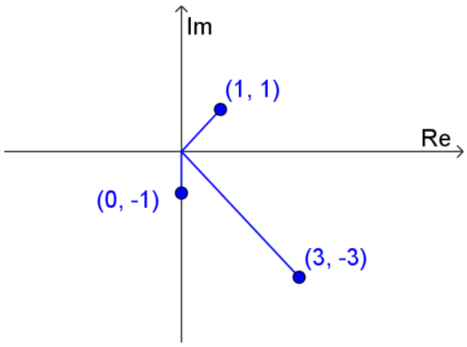
   $\left|\dfrac{z}{w}\right| = \left|\dfrac{-2-11i}{5}\right| = \dfrac{1}{5}|-2-11i|$

   $= \dfrac{1}{5}\sqrt{(-2)^2+(-11)^2} = \dfrac{1}{5}\sqrt{4+121} = \dfrac{1}{5}\sqrt{125} = \sqrt{5}$

2. (i) $\arg(1+i) = \dfrac{\pi}{4}$

   (ii) $\arg(-i) = -\dfrac{\pi}{2}$

   (iii) $\arg(3-3i) = -\dfrac{\pi}{4}$



3. (i) $|3+4i| = \sqrt{3^2+4^2} = 5$

   $3+4i$ is in the first quadrant, so $\arg(3+4i) = \tan^{-1}\frac{4}{3} = 0.927$ rad

   $3+4i = 5(\cos 0.927 + i\sin 0.927)$

   (ii) $|1-i| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

   $1-i$ is in the fourth quadrant, so $\arg(1-i) = -\tan^{-1}\frac{1}{1} = -\frac{\pi}{4}$

   $1-i = \sqrt{2}\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)$

   (iii) $\left|-\sqrt{3}-i\right| = \sqrt{(-\sqrt{3})^2+(-1)^2} = 2$

   $-\sqrt{3}-i$ is in the third quadrant, so $\arg(\sqrt{3}-i) = \tan^{-1}\frac{1}{\sqrt{3}} - \pi = -\frac{5\pi}{6}$

   $-\sqrt{3}-i = 2(\cos(-\frac{5\pi}{6}) + i\sin(-\frac{5\pi}{6}))$

Edexcel AS FM Complex numbers 1 Exercise solns

4. (i) $r = 3, \quad \theta = \frac{\pi}{4}$

$$x = r\cos\theta = 3\cos\frac{\pi}{4} = \frac{3}{\sqrt{2}}$$

$$y = r\sin\theta = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}}$$

$$z = \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$$

(ii) $r = 6, \quad \theta = \frac{2\pi}{3}$

$$x = r\cos\theta = 6\cos\left(\frac{2\pi}{3}\right) = 6 \times -\frac{1}{2} = -3$$

$$y = r\sin\theta = 6\sin\left(\frac{2\pi}{3}\right) = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

$$z = -3 + 3\sqrt{3}i$$

(iii) $r = 2, \quad \theta = -\frac{\pi}{6}$

$$x = r\cos\theta = 2\cos\left(-\frac{\pi}{6}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$y = r\sin\theta = 2\sin\left(-\frac{\pi}{6}\right) = 2 \times -\frac{1}{2} = -1$$

$$z = \sqrt{3} - i$$

5. $|z| = 2 \qquad \arg z = 1.2$

$|w| = 3 \qquad \arg w = 0.5$

(i) $|zw| = |z||w| = 2 \times 3 = 6$

$$\arg(zw) = \arg z + \arg w = 1.2 + 0.5 = 1.7$$

$$zw = 6(\cos 1.7 + i\sin 1.7)$$

(ii) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{2}{3}$

$$\arg\left(\frac{z}{w}\right) = \arg z - \arg w = 1.2 - 0.5 = 0.7$$

$$\frac{z}{w} = \tfrac{2}{3}(\cos 0.7 + i\sin 0.7)$$

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(iii) $\left|\frac{w}{z}\right| = \frac{|w|}{|z|} = \frac{3}{2}$

$\arg\left(\frac{w}{z}\right) = \arg w - \arg z = 0.5 - 1.2 = -0.7$

$\frac{w}{z} = \frac{3}{2}\left(\cos(-0.7) + i\sin(-0.7)\right)$

6. $|z| = 6 \qquad \arg z = \frac{5\pi}{6}$

$|w| = 4 \qquad \arg w = -\frac{\pi}{4}$

(i) $|zw| = |z||w| = 6 \times 4 = 24$

$\arg(zw) = \arg z + \arg w = \frac{5\pi}{6} + (-\frac{\pi}{4}) = \frac{7\pi}{12}$

$zw = 24\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

(ii) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{6}{4} = \frac{3}{2}$

$\arg z - \arg w = \frac{5\pi}{6} - (-\frac{\pi}{4}) = \frac{13\pi}{12}$

Since $\frac{13}{12}\pi$ does not lie in the range $-\pi < \theta \leq \pi$,

$\arg\frac{z}{w} = \frac{13}{12}\pi - 2\pi = -\frac{11}{12}\pi$

$\frac{z}{w} = \frac{3}{2}\left(\cos(-\frac{11\pi}{12}) + i\sin(-\frac{11\pi}{12})\right)$

(iii) $\left|\frac{w}{z}\right| = \frac{|w|}{|z|} = \frac{4}{6} = \frac{2}{3}$

$\arg w - \arg z = -\frac{\pi}{4} - \frac{5\pi}{6} = -\frac{13\pi}{12}$

Since $-\frac{13}{12}\pi$ does not lie in the range $-\pi < \theta \leq \pi$,

$\arg\frac{z}{w} = -\frac{13}{12}\pi + 2\pi = \frac{11}{12}\pi$

$\frac{z}{w} = \frac{2}{3}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$