Section 1: Introduction to complex numbers

Exercise level 3

1. $$\frac{1}{1-i}=\frac{1+i}{(1-i)(1+i)}=\frac{1+i}{1+1}=\tfrac{1}{2}(1+i)$$

$$1+\frac{1}{1-i}=1+\tfrac{1}{2}(1+i)=\tfrac{3}{2}+\tfrac{1}{2}i$$

$$\frac{1}{1+\frac{1}{1-i}}=\frac{1}{\frac{3}{2}+\frac{1}{2}i}=\frac{2}{3+i}=\frac{2(3-i)}{(3+i)(3-i)}=\frac{2(3-i)}{9+1}=\frac{3-i}{5}$$

$$1-\frac{1}{1+\frac{1}{1-i}}=1-\frac{3-i}{5}=\frac{5-3+i}{5}=\frac{2+i}{5}$$

$$\frac{1}{1-\frac{1}{1+\frac{1}{1-i}}}=\frac{5}{2+i}=\frac{5(2-i)}{(2+i)(2-i)}=\frac{5(2-i)}{4+1}=2-i.$$

2. (i) $(i+1)^2=i^2+2i+1=-1+2i+1=2i$

$(i+1)^3=2i(i+1)=2i^2+2i=-2+2i$

$(i+1)^4=(-2+2i)(i+1)=-2i+2i^2-2+2i=-2i-2-2+2i=-4$

(ii) $$(i-1)^4=i^4-4i^3+6i^2-4i+1$$
$$=1+4i-6-4i+1$$
$$=-4$$

$$(i+1)^{3600}-(i-1)^{3600}=\left((i+1)^4\right)^{900}-\left((i-1)^4\right)^{900}$$
$$=(-4)^{900}-(-4)^{900}$$
$$=0$$

3. (i) $i^2=-1$

$i^3=-j$

$i^4=(-1)^2=1$

(ii) The powers of i repeat every four terms.
Similarly, for negative powers of i:

$$i^{-1}=\frac{1}{i}=\frac{i}{i^2}=-i$$

$$i^{-2}=-i\times\frac{1}{i}=-1$$

$$i^{-3}=-1\times i=-\frac{i}{i^2}=i$$

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$i^{-4} = i \times \frac{1}{i} = 1$

So the negative powers of i also repeat every 4 terms, and therefore the values of $S = i^n + i^{-n}$ also repeat every 4 terms.

n	1	2	3	4
i^n	i	-1	$-i$	1
i^{-n}	$-i$	-1	i	1
$S = i^n + i^{-n}$	0	-2	0	2

So the possible values of S are 0, 2 and -2.